# PEARSON TEXAS ALGEBRA I

## STUDENT TEXT AND HOMEWORK HELPER

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ISBN-13: 978-0-13-330063-5
ISBN-10: 0-13-330063-3
1 2 3 4 5 6 7 8 9 10 V0YJ 20 19 18 17 16 15 14

## TOPIC 9 Exponential Functions and Equations



### TEXAS ESSENTIAL KNOWLEDGE AND SKILLS (TEKS) FOCUS

**(9)(A)** Determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

**(9)(B)** Interpret the meaning of the values of $a$ and $b$ in exponential functions of the form $f(x) = ab^x$ in real-world problems.

**(9)(C)** Write exponential functions in the form $f(x) = ab^x$ (where $b$ is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay.

**(9)(D)** Graph exponential functions that model growth and decay and identify key features, including $y$-intercept and asymptote, in mathematical and real-world problems.

**(9)(E)** Write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

**(12)(B)** Evaluate functions, expressed in function notation, given one or more elements in their domains.

# Topic 9 Exponential Functions and Equations

## TOPIC OVERVIEW



## VOCABULARY

**English/Spanish Vocabulary Audio Online:**

| English | Spanish |
|---|---|
| asymptote, *p. 386* | asíntota |
| compound interest, *p. 396* | interés compuesto |
| decay factor, *p. 396* | factor de decremento |
| exponential decay, *p. 396* | decremento exponencial |
| exponential function, *p. 386* | función exponencial |
| exponential growth, *p. 395* | incremento exponencial |
| growth factor, *p. 395* | factor incremental |

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# 9-1 Exponential Functions

## TEKS FOCUS

**TEKS (9)(A)** Determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

**TEKS (1)(C)** Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

**Additional TEKS (1)(A), (1)(D), (9)(C), (9)(D), (12)(B)**

## VOCABULARY

- **Asymptote** – An asymptote is a line that a graph of a function gets closer to as $x$ or $y$ gets larger in absolute value.
- **Exponential function** – An exponential function is a function of the form $y = ab^x$, where $a$ is a nonzero constant, $b > 0$, and $b \neq 1$.

- **Number sense** – the understanding of what numbers mean and how they are related

## ESSENTIAL UNDERSTANDING

Some functions model an initial amount that is repeatedly multiplied by the same positive number. In the rules for these functions, the independent variable is an exponent.

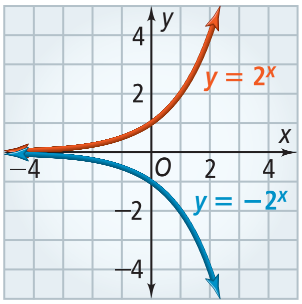
take note

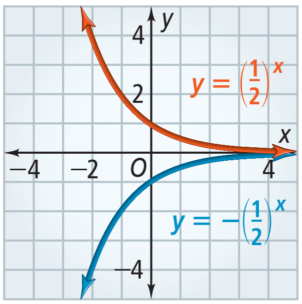
### Key Concept Exponential Function

**Definition**

An **exponential function** is a function of the form $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, $b \neq 1$, and $x$ is a real number.

**Examples**



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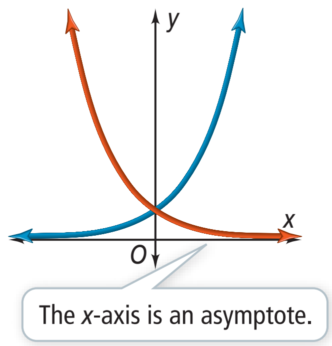
## Key Concept Asymptote of an Exponential Function $y = ab^x$

The $x$-axis ($y = 0$) is the horizontal asymptote for an exponential function of the form $y = ab^x$.

The $y$-values approach zero, but never actually reach zero. The graphs approach, but do not cross, the $x$-axis.




### Problem 1

### Identifying Linear and Exponential Functions

**Think**

**How can you identify a constant ratio between $y$-values?**
When you multiply each $y$-value by the same constant and get the next $y$-value, there is a constant ratio between the values.

**Does the table or rule represent a linear or an exponential function? Explain.**

**A**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | –1 | –3 | –9 | –27 |

The difference between the $x$-values is 1.

+1 +1 +1

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | –1 | –3 | –9 | –27 |

×3 ×3 ×3

The ratio between the $y$-values is 3.

The table represents an exponential function. There is a common difference between consecutive $x$-values and a common ratio between the corresponding $y$-values.

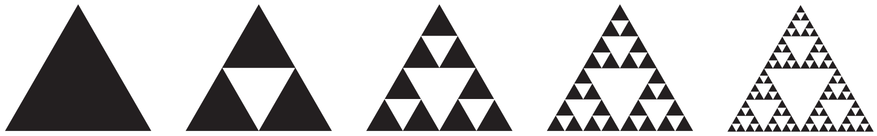
**B** $y = 3x$

The rule represents a linear function because it is in the form $y = mx + b$. The independent variable $x$ is not an exponent.

**Problem 2** TEKS Process Standard (1)(C)

## Writing an Exponential Function

**The figures below are stages of the *Sierpinski triangle*. In Stage 0, there is one black triangle. In each subsequent stage, a white triangle is added to the center of each black triangle, dividing that triangle into 3 black triangles and 1 white triangle. The number of black triangles increases by a factor of 3 in each stage.**




**A Write an exponential function $f(x) = ab^x$ to describe this situation.**

The factor by which the number of black triangles increases at each stage is the value of *b*. Using the fact that *b* is 3, you can show that the value of *a* is 1, the number of triangles in Stage 0.

| | |
|---|---|
| $f(x) = ab^x$ | Write the formula for the exponential function. |
| $f(x) = a \cdot 3^x$ | Substitute 3 for *b*. |
| $1 = a \cdot 3^0$ | From Stage 0, you know that $f(0) = 1$. |
| $1 = a \cdot 1$ | Simplify $3^0$. |
| $1 = a$ | Solve for *a*. |
| $f(x) = 1 \cdot 3^x$ | Substitute 1 for *a* in the formula $f(x) = a \cdot 3^x$. |
| $f(x) = 3^x$ | Simplify. |

**Think**

**Is there any stage with exactly 1000 black triangles?**
No. At each stage the number of black triangles will be a multiple of 3.

**B Determine in what stage there will first be more than 1000 black triangles.**

You can select techniques such as *estimation* and *mental math* to solve this problem.

In Stage 2, you know there are 9 black triangles. Round 9 to 10 and then multiply by the factor 3 to estimate the number of black triangles in later stages.

| Stage | Triangle Estimate |
|---|---|
| 2 | 10 |
| 3 | 30 |
| 4 | 100 |
| 5 | 300 |
| 6 | 900 |

$30 \cdot 3 = 90$
Now round 90 to 100.

The estimate of 900 in Stage 6 is high, since you rounded up in earlier stages. So, you know the number of black triangles in Stage 6 is less than 1000. When you multiply by 3 again, the number of black triangles will exceed 1000.

There will first be more than 1000 black triangles in Stage 7.

**Check** You can confirm this result by evaluating $f(6)$ and $f(7)$.

$f(x) = 3^6 = 729$ $\qquad$ $f(x) = 3^7 = 2187$

## Problem 3

TEKS Process Standard (1)(A)

### Think

**Why is the function $30 \cdot 2^x$ not $2 \cdot 30^x$?**
In an exponential function, $a$ is the starting value and $b$ is the common ratio.

### Evaluating an Exponential Function

**Population Growth** **Suppose 30 flour beetles are left undisturbed in a warehouse bin. The beetle population doubles each week. The function $f(x) = 30 \cdot 2^x$ gives the population after $x$ weeks. How many beetles will there be after 56 days?**

$f(x) = 30 \cdot 2^x$

$= 30 \cdot 2^8$ 56 days is equal to 8 weeks. Evaluate the function for $x = 8$.

$= 30 \cdot 256$ Simplify the power.

$= 7680$ Simplify.

After 56 days, there will be 7680 beetles.

## Problem 4

TEKS Process Standard (1)(D)

### Think

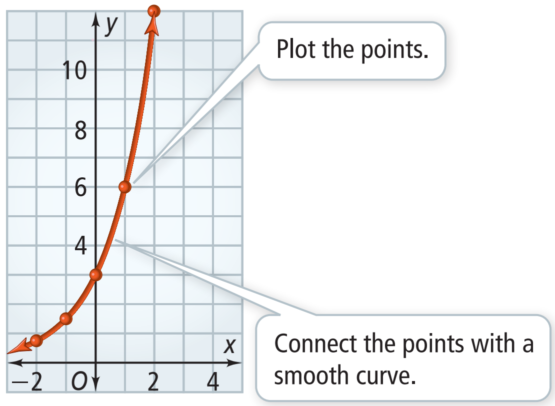
**How can you use the graph to find the domain and range?**
The domain can be found by using the $x$-coordinates on the graph and the range can be found by using the $y$-coordinates on the graph.

### Graphing an Exponential Function

**A** **What is the graph of $y = 3 \cdot 2^x$? Identify the domain and range. Express the range using an inequality.**

Make a table of $x$- and $y$-values.

| $x$ | $y = 3 \cdot 2^x$ | $(x, y)$ |
|---|---|---|
| $-2$ | $3 \cdot 2^{-2} = \frac{3}{2^2} = \frac{3}{4}$ | $\left(-2, \frac{3}{4}\right)$ |
| $-1$ | $3 \cdot 2^{-1} = \frac{3}{2^1} = 1\frac{1}{2}$ | $\left(-1, 1\frac{1}{2}\right)$ |
| 0 | $3 \cdot 2^0 = 3 \cdot 1 = 3$ | (0, 3) |
| 1 | $3 \cdot 2^1 = 3 \cdot 2 = 6$ | (1, 6) |
| 2 | $3 \cdot 2^2 = 3 \cdot 4 = 12$ | (2, 12) |




Any value substituted for $x$ results in a positive $y$-value.

The domain is all real numbers. The range is $y > 0$.

**B** **What is the $y$-intercept of the exponential function in part (A)? What is the asymptote?**

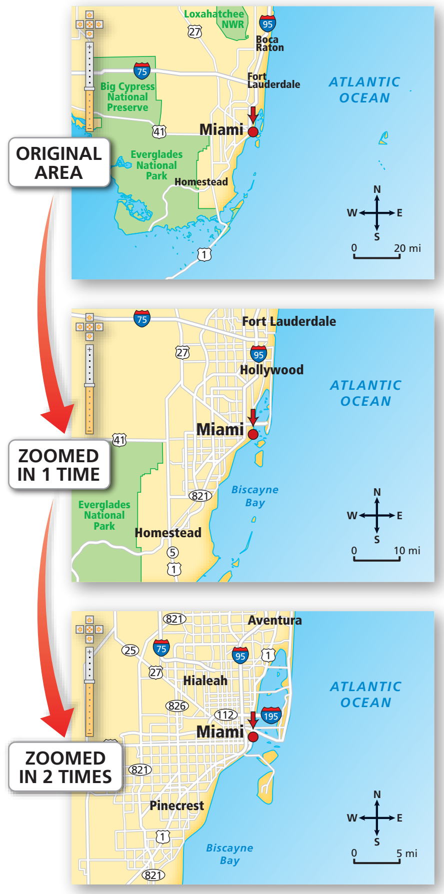
The table and the graph show that when $x = 0$, $y = 3$. The $y$-intercept of $y = 3 \cdot 2^x$ is 3.

The table and the graph show that as $x$ decreases, the $y$-values get smaller. The value of $3 \cdot 2^x$ will always be positive, so the $y$-values will approach zero, but never reach zero. The $x$-axis is the horizontal asymptote for $y = 3 \cdot 2^x$.

## Problem 5

### Graphing an Exponential Model

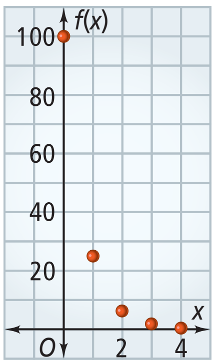
**Maps** **Computer mapping software allows you to zoom in on an area to view it in more detail. The function $f(x) = 100 \cdot 0.25^x$ models the percent of the original area the map shows after zooming in $x$ times. Graph the function. Identify the $y$-intercept, the asymptote, and the domain.**




| $x$ | $f(x) = 100 \cdot 0.25^x$ | $(x, f(x))$ |
|---|---|---|
| 0 | $100 \cdot 0.25^0 = 100$ | (0, 100) |
| 1 | $100 \cdot 0.25^1 = 25$ | (1, 25) |
| 2 | $100 \cdot 0.25^2 = 6.25$ | (2, 6.25) |
| 3 | $100 \cdot 0.25^3 \approx 1.56$ | (3, 1.56) |
| 4 | $100 \cdot 0.25^4 \approx 0.39$ | (4, 0.39) |

**Think**

**Should you connect the points of the graph?**
No, the data is discrete not continuous. The number of times you zoom in must be a nonnegative integer.




When $x = 0$, $y = 100$, so the $y$-intercept is 100. The $x$-axis is the horizontal asymptote. The domain is $x \geq 0$, $x$ is an integer.

## Problem 6

TEKS Process Standard (1)(C)

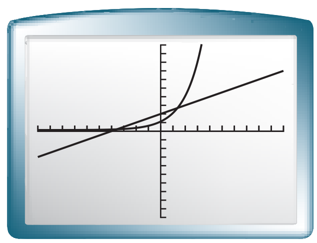
### Solving One-Variable Equations

**What is the solution or solutions of $2^x = 0.5x + 2$?**

**Step 1** Write each side of the equation as a function equation.

$f(x) = 2^x$ and $g(x) = 0.5x + 2$

**Step 2** Graph the equations using a graphing calculator. Use $y_1$ for $f(x)$ and $y_2$ for $g(x)$.



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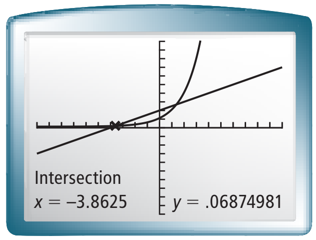
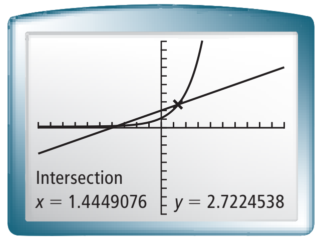
## Problem 6 *continued*

**Step 3** Use the **CALC** feature. Chose **INTERSECT** to find the points where the lines intersect.




The solutions of $2^x = 0.5x + 2$ are about $-3.86$ and $1.44$.

**Think**

**How can you check that the $x$-value is a solution?**
Substitute for $x$ in the original equation. Make sure you use the same $x$-value for each instance of $x$.

## PRACTICE and APPLICATION EXERCISES

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**Determine whether each table or rule represents a linear or an exponential function. Explain why or why not.**

**1.**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 2 | 8 | 32 | 128 |

**2.**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 6 | 9 | 12 | 15 |

**3.** $y = 4 \cdot 5^x$ **4.** $y = 12 \cdot x$ **5.** $y = -5 \cdot 0.25^x$ **6.** $y = 7x + 3$

**Evaluate each function for the given value.**

**7.** $f(x) = 6^x$ for $x = 2$

**8.** $g(t) = 2 \cdot 0.4^t$ for $t = -2$

**9.** $y = 20 \cdot 0.5^x$ for $x = 3$

**10.** $h(w) = -0.5 \cdot 4^w$ for $w = 18$

**11.** **Apply Mathematics (1)(A)** An investment of \$5000 doubles in value every decade. The function $f(x) = 5000 \cdot 2^x$, where $x$ is the number of decades, models the growth of the value of the investment. How much is the investment worth after 30 yr?

**12.** **Apply Mathematics (1)(A)** A population of 75 foxes in a wildlife preserve quadruples in size every 15 yr. The function $y = 75 \cdot 4^x$, where $x$ is the number of 15-yr periods, models the population growth. How many foxes will there be after 45 yr?

**Graph each exponential function. Identify the domain, range, $y$-intercept, and asymptote of each function. Express each range using an inequality.**

**13.** $y = 4^x$ **14.** $y = -4^x$ **15.** $y = \left(\frac{1}{3}\right)^x$ **16.** $y = -\left(\frac{1}{3}\right)^x$

**17.** $y = 10 \cdot \left(\frac{3}{2}\right)^x$ **18.** $y = 0.1 \cdot 2^x$ **19.** $y = \frac{1}{4} \cdot 2^x$ **20.** $y = 1.25^x$

**Select Tools to Solve Problems (1)(C)** Use a graph to solve each equation.

**21.** $4^x = \frac{3}{2}x + 5$

**22.** $x + 3 = 3^x$

**23.** **Analyze Mathematical Relationships (1)(F)** A new museum had 7500 visitors this year. The museum curators expect the number of visitors to grow by 5% each year. The function $y = 7500 \cdot 1.05^x$ models the predicted number of visitors each year after $x$ years. Graph the function. Identify the domain, range, $y$-intercept, and asymptote of the function. Express the range using an inequality.

**24.** **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** A solid waste disposal plan proposes to reduce the amount of garbage each person throws out by 2% each year. This year, each person threw out an average of 1500 lb of garbage. The function $y = 1500 \cdot 0.98^x$ models the average amount of garbage each person will throw out each year after $x$ years. Graph the function. Identify the domain, range, $y$-intercept, and asymptote of the function. Express the range using an inequality.

**25.** **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** Compare the rule and the function table below. Which function has the greater value when $x = 12$? Explain.

**Function 1**

$y = 4^x$

**Function 2**

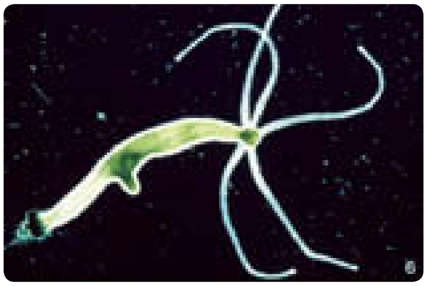
| | | | | |
|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 |
| $y$ | 5 | 25 | 125 | 625 |

**26.** You have just read a journal article about a population of fungi that doubles every 3 weeks. The beginning population was 10. The function $y = 10 \cdot 2^{\frac{n}{3}}$ represents the population after $n$ weeks.

**a.** You have a population of 15 of the same fungi. Assuming the journal article gives the correct rate of increase, write the function that represents the population of fungi after $n$ weeks.

**b.** Suppose you find another article that states that the fungi population triples every 4 weeks. If there are currently 15 fungi in your population, write the function that represents the population after $n$ weeks.

**27.** **Explain Mathematical Ideas (1)(G)** Hydra are small freshwater animals. They can double in number every two days in a laboratory tank. Suppose one tank has an initial population of 60 hydra. When will there be more than 5000 hydra?


Hydra

**28. a.** Graph $y = 2^x$, $y = 4^x$, and $y = 0.25^x$ on the same axes.

**b.** What point is on all three graphs?

**c.** Does the graph of an exponential function intersect the $x$-axis? Explain.

**d.** How does the graph of $y = b^x$ change as the base $b$ increases or decreases?

**Write an exponential function to describe the given sequence of numbers.**

**29.** 2, 8, 32, 128, 512, . . .

**30.** $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

**Which function has the greater value for the given value of *x*?**

**31.** $y = 4^x$ or $y = x^4$ for $x = 2$

**32.** $f(x) = 10 \cdot 2^x$ or $f(x) = 200 \cdot x^2$ for $x = 7$

**33.** $y = 3^x$ or $y = x^3$ for $x = 5$

**34.** $f(x) = 2^x$ or $f(x) = 100x^2$ for $x = 10$

**35. Apply Mathematics (1)(A)** A computer valued at $1500 loses 20% of its value each year.

**a.** Write a function rule that models the value of the computer.

**b.** Find the value of the computer after 3 yr.

**c.** In how many years will the value of the computer be less than $500?

**36. a.** Graph the functions $y = x^2$ and $y = 2^x$ on the same axes.

**b.** What do you notice about the graphs for the values of *x* between 1 and 3?

**c.** How do you think the graph of $y = 8^x$ would compare to the graphs of $y = x^2$ and $y = 2^x$?

**37. Explain Mathematical Ideas (1)(G)** Find the range of the function $f(x) = 500 \cdot 1^x$ using the domain $\{1, 2, 3, 4, 5\}$. Explain why the definition of *exponential function* states that $b \neq 1$.

**Evaluate each function over the domain $\{-2, -1, 0, 1, 2, 3\}$. As the values of the domain increase, do the values of the range *increase* or *decrease*?**

**38.** $f(x) = 5^x$

**39.** $y = 2.5^x$

**40.** $h(x) = 0.1^x$

**Solve each equation.**

**41.** $2^x = 64$

**42.** $3^x = \frac{1}{27}$

**43.** $3 \cdot 2^x = 24$

**44.** $5 \cdot 2^x - 152 = 8$

**45.** Suppose (0, 4) and (2, 36) are on the graph of an exponential function.

**a.** Use (0, 4) in the general form of an exponential function, $y = a \cdot b^x$, to find the value of the constant $a$.

**b.** Use your answer from part (a) and (2, 36) to find the value of the constant $b$.

**c.** Write a rule for the function.

**d.** Evaluate the function for $x = -2$ and $x = 4$.

**46.** A population of 150 dandelions is growing in a meadow. A computer model predicts that the population will increase by a factor of 1.2 every week. Write an equation to show the population as a function of time *x*, in which *x* is the number of weeks. What will be the approximate population after 4 weeks?

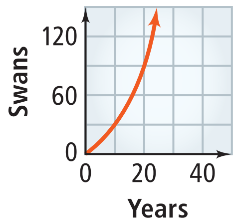
47. A bank offers a special account in which they accept an initial deposit of 3 cents, and then double the value of the account every month.

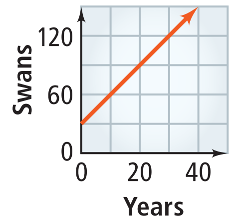
a. Write an equation to show the value of the account in dollars as a function of $x$ months. What is the value in the account after 3 months?

b. **Select Techniques to Solve Problems (1)(C)** Select a technique such as number sense, mental math, or estimation to determine when the value of the account will be more than \$15.

## TEXAS End-of-Course PRACTICE

48. A population of 30 swans doubles every 10 yr. Which graph represents the population growth?

A.



C.
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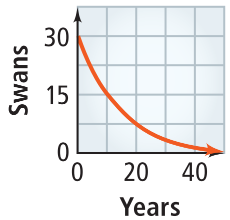
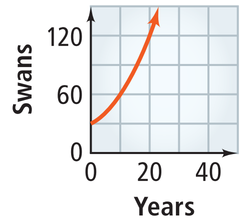



D.



49. Which equation do you get when you solve $y = 2x - 12$ for $x$?

F. $x = y - 6$

G. $x = y + 6$

H. $x = 0.5y - 6$

J. $x = 0.5y + 6$

# 9-2 Exponential Growth and Decay

## TEKS FOCUS

**TEKS (9)(B)** Interpret the meaning of the values of $a$ and $b$ in exponential functions of the form $f(x) = ab^x$ in real-world problems.

**TEKS (1)(A)** **Apply** mathematics to problems arising in everyday life, society, and the workplace.

**Additional TEKS (1)(G), (9)(C), (9)(D)**

## VOCABULARY

- **Compound interest** – interest paid on both the principal and the interest that has already been paid
- **Decay factor** – 1 minus the percent rate of change expressed as a decimal for an exponential decay situation; the base $b$ in the function $y = a \cdot b^x$
- **Exponential decay** – a situation that can be modeled with a function of the form $y = a \cdot b^x$, where $a > 0$ and $0 < b < 1$
- **Exponential growth** – a situation that can be modeled with a function of the form $y = a \cdot b^x$, where $a > 0$ and $b > 1$
- **Growth factor** – 1 plus the percent rate of change expressed as a decimal for an exponential growth situation; the base $b$ in the function $y = a \cdot b^x$

---

- **Apply** – use knowledge or information for a specific purpose, such as solving a problem

## ESSENTIAL UNDERSTANDING

An exponential function can model growth or decay of an initial amount.

take note

## Key Concept Exponential Growth

**Definitions**

**Exponential growth** can be modeled by the function $y = a \cdot b^x$, where $a > 0$ and $b > 1$. The base $b$ is the **growth factor,** which equals 1 plus the percent rate of change expressed as a decimal.

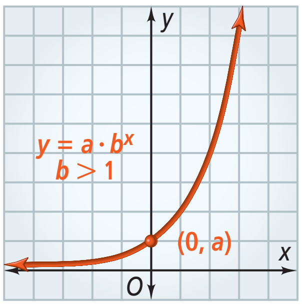
**Algebra**

The initial amount (when $x = 0$) is also the $y$-intercept.

$$y = a \cdot b^x \leftarrow \text{exponent}$$

The base, which is greater than 1, is the growth factor.

**Graph**



take note

## Key Concept Compound Interest

When a bank pays interest on both the principal *and* the interest an account has already earned, the bank is paying **compound interest.** You can use the following formula to find the balance of an account that earns compound interest.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A =$ the balance
$P =$ the principal (the initial deposit)
$r =$ the annual interest rate (expressed as a decimal)
$n =$ the number of times interest is compounded per year
$t =$ the time in years

take note

## Key Concept Exponential Decay

**Definitions**

**Exponential decay** can be modeled by the function $y = a \cdot b^x$, where $a > 0$ and $0 < b < 1$. The base $b$ is the **decay factor,** which equals 1 minus the percent rate of change expressed as a decimal.

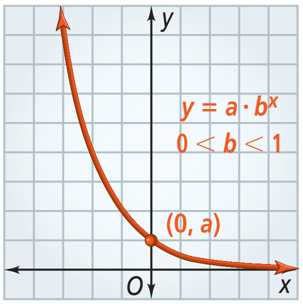
**Algebra**

The initial amount (when $x = 0$) is also the $y$-intercept.

$$y = a \cdot b^x \leftarrow \text{exponent}$$

The base is the decay factor.

**Graph**



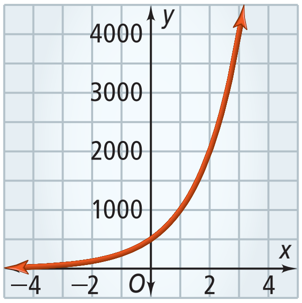

## Problem 1

### Graphing an Exponential Growth Function

**You are observing a bacteria population in a laboratory culture. The population doubles every 30 min. The function $p(x) = 500 \cdot 2^x$ models the population, where $x$ is the number of 30-minute periods since you began observing.**

**A Graph the function $p(x) = 500 \cdot 2^x$, identifying the $y$-intercept and asymptote.**

The equation $p(x) = 500 \cdot 2^x$ is in the form $y = a \cdot b^x$, where $a = 500$ and $b = 2$. Since $b > 1$, the function represents exponential growth. The graph rises from left to right and has a $y$-intercept of 500. The asymptote is the $x$-axis, with the graph approaching the $x$-axis as $x$ decreases.




**B What is the meaning of the $y$-intercept in this situation?**

The $y$-intercept 500 occurs when $x$ is 0, which corresponds to the time you start observing the population. There are 500 bacteria present when you start observing.

**Think**

**Does the point $(-1, 250)$ have meaning in this situation?**
$x = -1$ corresponds to the time 30 min before you start observing the bacteria population. Thirty minutes before you start, the number of bacteria is 250.

## Problem 2

TEKS Process Standard (1)(A)

### Modeling Exponential Growth

**Economics** **Since 2005, the amount of money spent at restaurants in the United States has increased about 7% each year. In 2005, about \$360 billion was spent at restaurants.**

**Think**

**When can you use an exponential growth function?**
You can use an exponential growth function when an initial amount increases by a fixed percent each time period.

**A** **If the trend continues, about how much will be spent at restaurants in 2015?**

**Relate** $y = a \cdot b^x$ Use an exponential function.

**Define** Let $x =$ the number of years since 2005.

Let $y =$ the annual amount spent at restaurants (in billions of dollars).

Let $a =$ the initial amount spent (in billions of dollars), 360.

Let $b =$ the growth factor, which is $1 + 0.07 = 1.07$.

**Write** $y = 360 \cdot 1.07^x$

Use the equation to predict the annual spending in 2015.

| | |
|---|---|
| $y = 360 \cdot 1.07^x$ | |
| $= 360 \cdot 1.07^{10}$ | 2015 is 10 yr after 2005, so substitute 10 for $x$. |
| $\approx 708$ | Round to the nearest billion dollars. |

About \$708 billion will be spent at restaurants in 2015 if the trend continues.

**B** **What is an expression that represents the equivalent monthly increase of spending at U.S. restaurants in 2005?**

You will need to find an expression of the form $r^m$, where $r$ is approximately the monthly growth factor and $m$ is the number of months. You know that $1.07^x$ represents the yearly increase where $x$ is the number of years.

| | |
|---|---|
| $1.07^x = 1.07^{\frac{12x}{12}}$ | There are $12x$ months in $x$ years. |
| $= \left(1.07^{\frac{1}{12}}\right)^{12x}$ | Power raised to a power |
| $\approx 1.0057^{12x}$ | Simplify. |
| $= 1.0057^m$ | Let $12x = m$, the number of months. |

The expression $1.0057^m$ represents the equivalent monthly increase of spending.

## Problem 3

### Compound Interest

**Finance** **Suppose that when your friend was born, your friend's parents deposited \$2000 in an account paying 4.5% interest compounded quarterly. What will the account balance be after 18 yr?**

**Think**

**Is the formula an exponential growth function?**
Yes. You can rewrite the formula as $A = P\left[\left(1+\frac{r}{n}\right)^n\right]^t$. So, it is an exponential function with initial amount $P$ and growth factor $\left(1+\frac{r}{n}\right)^n$.

| Know | Need | Plan |
|---|---|---|
| • \$2000 principal<br>• 4.5% interest<br>• interest compounded quarterly | Account balance in 18 yr | Use the compound interest formula. |

$A = P\left(1+\frac{r}{n}\right)^{nt}$ — Use the compound interest formula.

$= 2000\left(1+\frac{0.045}{4}\right)^{4 \cdot 18}$ — Substitute the values for $P$, $r$, $n$, and $t$.

$= 2000(1.01125)^{72}$ — Simplify.

The balance will be \$4475.53 after 18 yr.

## Problem 4

### Writing an Exponential Decay Function

**Think**

**How do the equations for exponential growth and decay functions compare?**
The general equation for both functions is $f(x) = ab^x$, but $b > 1$ for growth, and $0 < b < 1$ for decay.

**The value of a function decreases by a factor of 0.3 with every unit increase in $x$. When $x$ is 0, the value of the function is 12. Write an exponential function in the form $f(x) = ab^x$ to describe the situation.**

$f(x) = ab^x$ — Write the equation for exponential decay.

$f(x) = 12 \cdot b^x$ — Substitute 12 for $a$, the initial amount or $f(0)$.

$f(x) = 12 \cdot 0.3^x$ — Substitute 0.3 for $b$, the factor by which $f(x)$ decreases for every unit increase in $x$.

The function that describes the situation is $f(x) = 12 \cdot 0.3^x$.

## Problem 5

### Graphing an Exponential Decay Function

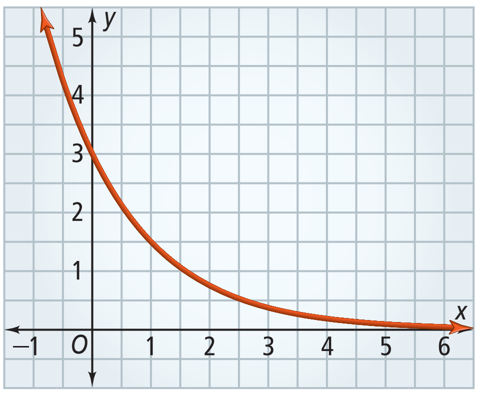
**Graph the function $y = 3 \cdot 0.5^x$. Identify the $y$-intercept and the asymptote.**

The equation $y = 3 \cdot 0.5^x$ is in the form $y = a \cdot b^x$, where $a = 3$ and $b = 0.5$.

Since $0 < b < 1$, the function represents exponential decay.

The graph falls from left to right and has a $y$-intercept of 3.

The asymptote is the $x$-axis, with the graph approaching the $x$-axis as $x$ increases.



## Problem 6

TEKS Process Standard (1)(G)

### Modeling Exponential Decay STEM

**Physics** **The kilopascal is a unit of measure for atmospheric pressure. The atmospheric pressure at sea level is about 101 kilopascals. For every 1000-m increase in altitude, the pressure decreases about 11.5%. What is the approximate pressure at an altitude of 3000 m?**

**Think**

**Will the pressure ever be negative?**
No. The range of an exponential decay function is all positive real numbers. The graph of an exponential decay function approaches but does not cross the $x$-axis.

**Relate** $y = a \cdot b^x$ Use an exponential function.

**Define** Let $x$ = the altitude (in thousands of meters).

Let $y$ = the atmospheric pressure (in kilopascals).

Let $a$ = the initial pressure (in kilopascals), 101.

Let $b$ = the decay factor, which is $1 - 0.115 = 0.885$.

**Write** $y = 101 \cdot 0.885^x$

Use the equation to estimate the pressure at an altitude of 3000 m.

$y = 101 \cdot 0.885^x$

$= 101 \cdot 0.885^3$ Substitute 3 for $x$.

$\approx 70$ Round to the nearest kilopascal.

The pressure at an altitude of 3000 m is about 70 kilopascals.

## ONLINE HOMEWORK PRACTICE and APPLICATION EXERCISES

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For additional support when completing your homework, go to **PearsonTEXAS.com**.

**1. Apply Mathematics (1)(A)** The value of a stock purchase increases by a factor of 1.5 every year. The value of the stock $y$, in dollars, as a function of time $x$, in years, is given by the equation $y = 100(1.5)^x$. Graph this function and identify the $y$-intercept and the asymptote. What does the $y$-intercept show about the stock?

**2. Apply Mathematics (1)(A)** A population of strangler figs, a type of vine, was established in the forest many years ago. The total length $y$ of the vines, in feet, over time $x$, in years, is modeled by the equation $y = 12(2)^{\frac{x}{5}}$. The value $x = 0$ represents the present. Graph this function. What are the $y$-intercept and the asymptote of the graph? What does the $y$-intercept of the graph represent?

**3. Explain Mathematical Ideas (1)(G)** An exponential function has the form $y = a(b)^{kx}$, where $a$ and $b$ are positive and $b \neq 1$. Without graphing, how do you know whether the function will model exponential growth or decay? Explain your answer.

**Identify the initial amount $a$ and the growth factor $b$ in each exponential function.**

**4.** $g(x) = 14 \cdot 2^x$

**5.** $y = 150 \cdot 1.0894^x$

**6.** $y = 25{,}600 \cdot 1.01^x$

**7.** $f(t) = 1.4^t$

8. The number of students enrolled at a college is 15,000 and grows 4% each year.
   a. The initial amount $a$ is ▆.
   b. The percent rate of change is 4%, so the growth factor $b$ is $1 +$ ▆ $=$ ▆.
   c. To find the number of students enrolled after one year, you calculate $15{,}000 \cdot$ ▆.
   d. Complete the equation $y =$ ▆ $\cdot$ ▆$^{▆}$ to find the number of students enrolled after $x$ years.
   e. Use your equation to predict the number of students enrolled after 25 yr.
9. **Apply Mathematics (1)(A)** A population of 100 frogs increases at an annual rate of 22%. How many frogs will there be in 5 years? Write an expression to represent the equivalent monthly population increase rate.

**Find the balance in each account after the given period.**

10. \$4000 principal earning 6% compounded annually, after 5 yr
11. \$12,000 principal earning 4.8% compounded annually, after 7 yr
12. \$500 principal earning 4% compounded quarterly, after 6 yr
13. \$5000 deposit earning 1.5% compounded quarterly, after 3 yr
14. \$775 deposit earning 4.25% compounded annually, after 12 yr
15. \$3500 deposit earning 6.75% compounded monthly, after 6 months

**Write the exponential function that describes each situation.**

16. The value of the function decreases by a factor of $\frac{1}{2}$ with each unit increase in $x$. When $x = 0$, the value of the function is 42.
17. The initial amount of the function is 12, and then it decreases by a factor of $\frac{4}{5}$ with each unit increase in $x$.
18. The initial value of the function is 3000, and then it loses 20% of its value with each unit increase in $x$.
19. **Explain Mathematical Ideas (1)(G)** The graph of an exponential function passes through the points (3, 100) and (5, 25). Write a function $f(x) = ab^x$ that describes this situation. Explain your reasoning.
20. **Apply Mathematics (1)(A)** A family buys a car for \$20,000. The value of the car decreases about 20% each year. After 6 yr, the family decides to sell the car. Should they sell it for \$4000? Explain.
21. **Apply Mathematics (1)(A)** You invest \$100 and expect your money to grow 8% each year. About how many years will it take for your investment to double?
22. **Display Mathematical Ideas (1)(G)** Suppose you start a lawn-mowing business and make a profit of \$400 in the first year. Each year, your profit increases 5%.
   a. Write a function that models your annual profit.
   b. If you continue your business for 10 yr, what will your *total* profit be?

**Graph the function. Identify the $y$-intercept and the asymptote.**

**23.** $f(x) = 3\left(\frac{1}{2}\right)^x$ **24.** $f(x) = 0.9\left(\frac{2}{3}\right)^x$ **25.** $f(x) = 2(0.1)^x + 1$

**26. Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** The graph of an exponential function is decreasing. The $y$-intercept is 300 and the range of the function is $y > 60$. Sketch a graph of the function, and write a possible equation for the function.

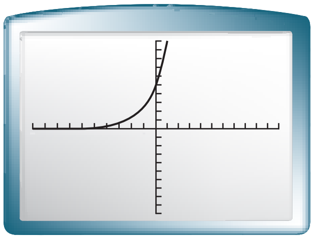
**Identify the initial amount *a* and the decay factor *b* in each exponential function.**

**27.** $y = 5 \cdot 0.5^x$ **28.** $f(x) = 10 \cdot 0.1^x$ **29.** $g(x) = 100\left(\frac{2}{3}\right)^x$ **30.** $y = 0.1 \cdot 0.9^x$

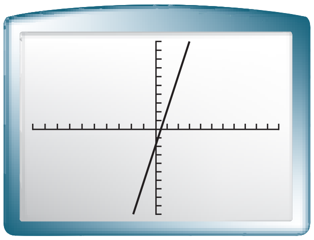
**31. Apply Mathematics (1)(A)** The population of a city is 45,000 and decreases 2% each year. If the trend continues, what will the population be after 15 yr?

**State whether each graph shows an *exponential growth function*, an *exponential decay function*, or *neither*.**

**32.**

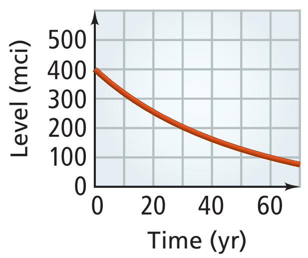


**33.**



STEM **34. Apply Mathematics (1)(A)** Radioactive decay is an example of exponential decay. The half-life of a radioactive substance is the length of time it takes for half of the atoms in a sample of the substance to decay. Cesium-137 is a radioisotope used in radiology where levels are measured in millicuries (mci). Use the graph at the right. Write the equation for the graph of Cesium-137 decay. What is a reasonable estimate of the half-life of cesium-137? What is the asymptote of the graph? Explain what it represents.

**Cesium-137 Decay**




## TEXAS End-of-Course PRACTICE

**35.** A new fitness center opens with 120 members. Every month the fitness center increases the number of members by 40 members. How many members will the fitness center have after being open for 8 months?

**36.** What is the slope of the line that passes through the points $(1, 1)$ and $(2, -1)$?

**37.** What is the simplified form of $5^{-2}$?

# 9-3 Modeling Exponential Data

## TEKS FOCUS

**TEKS (9)(E)** Write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

**TEKS (1)(E)** Create and use **representations** to organize, record, and communicate mathematical ideas.

**Additional TEKS (1)(C), (9)(A), (9)(B), (9)(C), (9)(D)**

## VOCABULARY

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

## ESSENTIAL UNDERSTANDING

You can use exponential functions to model some sets of data. When more than three data points suggest an exponential function, you can use the regression feature of a graphing calculator to find an exponential model.

## Problem 1

TEKS Process Standard (1)(E)

### Modeling Real-World Data

**Transportation** **The data at the right give the value of a used car over time. Which type of function best models the data? Write an equation to model the data.**

**Value of Used Car**

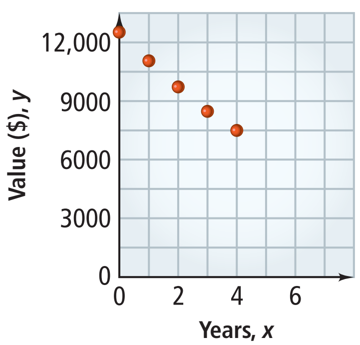
| Years | Value ($) |
|---|---|
| 0 | 12,575 |
| 1 | 11,065 |
| 2 | 9750 |
| 3 | 8520 |
| 4 | 7540 |

**Know**
The value of a used car over time

**Need**
The most appropriate model for the data

**Plan**
Graph the data and then use differences or ratios to find a model for the situation.

**Step 1**
Graph the data.




The graph curves and does not look quadratic. It may be exponential.

**Step 2**
Test for a common ratio.

| | Years | Value ($) | |
|---|---|---|---|
| | 0 | 12,575 | |
| +1 | 1 | 11,065 | $\frac{11,065}{12,575} \approx 0.88$ |
| +1 | 2 | 9750 | $\frac{9750}{11,065} \approx 0.88$ |
| +1 | 3 | 8520 | $\frac{8520}{9750} \approx 0.87$ |
| +1 | 4 | 7540 | $\frac{7540}{8520} \approx 0.88$ |

The value of the car is roughly 0.88 times its value the previous year.

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## Problem 1 *continued*

**Step 3**
Write an exponential model.

**Relate** $f(x) = a \cdot b^x$

**Define** Let $a =$ the initial value, 12,575.
Let $b =$ the decay factor, 0.88.

**Write** $f(x) = 12{,}575 \cdot 0.88^x$

**Step 4**
Test two points other than (0, 12,575).

Test (2, 9750):

$y = 12{,}575 \cdot 0.88^2$
$y \approx 9738$

Test (4, 7540):

$y = 12{,}575 \cdot 0.88^4$
$y \approx 7541$

The point (2, 9738) is close to the data point (2, 9750). The point (4, 7541) is close to the data point (4, 7540). The equation $y = 12{,}575 \cdot 0.88^x$ models the data.

## Problem 2

TEKS Process Standard (1)(C)

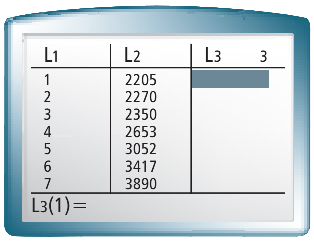
### Modeling Exponential Data Using Technology

**The table shows the numbers of bacteria in a culture after the given numbers of hours. Predict when the number of bacteria will reach 10,000.**

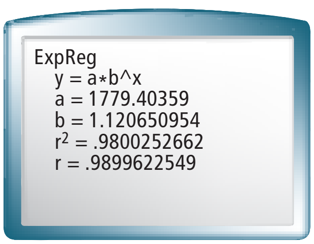
| Hour | Bacteria |
|---|---|
| 1 | 2205 |
| 2 | 2270 |
| 3 | 2350 |
| 4 | 2653 |
| 5 | 3052 |
| 6 | 3417 |
| 7 | 3890 |
| 8 | 4522 |
| 9 | 5107 |
| 10 | 5724 |

You can select technology as a tool to help you solve this problem. Using a graphing calculator, you can determine whether an exponential function is a reasonable model.

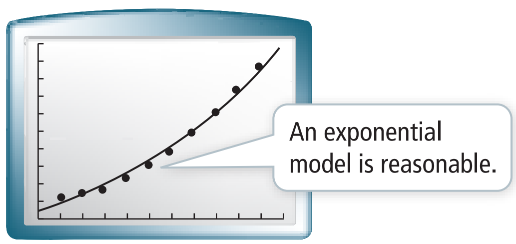
**Step 1** Enter the data.




**Step 2** Use **ExpReg**. The model is $f(x) = 1779.404(1.121)^x$.




**Step 3** Graph the data and the function to confirm the model.

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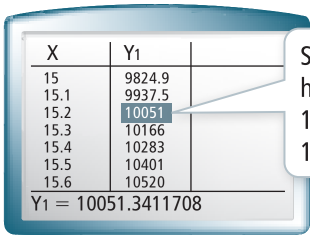
## Problem 2 *continued*

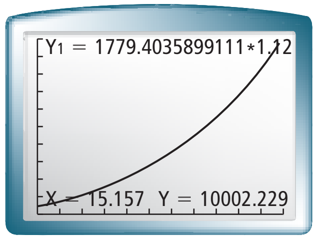
**Think**

**How can I see *x*-values that are closer together in the table?**
Go into the table setup for your calculator. You can change the increment between successive *x*-values.

**Step 4** Use the model to predict how long it will take for the number of bacteria to reach 10,000. Use tables or the trace feature on the graph. You may need to adjust the viewing window.




Since a tenth of an hour is 6 minutes, 15.2 hours means 15 hours 12 minutes.




The number of bacteria will reach 10,000 at about 15 hours 12 minutes.

## Problem 3

### Analyzing Features of Exponential Data

**A ball is dropped from a height of 75 in. The heights of the ball's rebounds after subsequent bounces are shown in the table.**

| Ball Rebounds | | | | | |
|---|---|---|---|---|---|
| Bounce | 0 | 1 | 2 | 3 | 4 |
| Height of Rebound (in.) | 75 | 46 | 29 | 18 | 11 |

**A What is an exponential model for this data?**

Enter the data into lists in a graphing calculator, and perform an exponential regression on the data. An exponential model for the data is $f(x) = 75 \cdot 0.62^x$.

**Think**

**How does the domain for the real-world data differ from the domain for the function?**
In the function, *x* could be any real number. The bounce number, however, must come from the set {0, 1, 2, …}. So the domain is the set of whole numbers.

**B Identify the domain and range of the function in part (A) in the context of the problem. Express the domain and range using inequalities.**

The domain is the bounce number. The bounce number cannot be negative, so the domain is $x \geq 0$, where $x$ is an integer. The range is the height of the rebound. The height starts at 75 in. and then decreases, but cannot be negative. It will get closer and closer to 0, but never reach 0 according to the model. The range is $0 < f(x) \leq 75$, where $f(x) = 75 \cdot 0.62^x$.

**C What is the *y*-intercept of the function in part (A), and what does it mean in the context of the problem?**

The $y$-intercept is the $y$-value when $x = 0$. When $x = 0$, $f(x) = 75 \cdot 0.62^0 = 75$, so the $y$-intercept is 75. In the context of the problem, this is the height of the ball after drop 0, or the initial height of the ball, 75 in.

**D An exponential function is a function in the form $f(x) = a \cdot b^x$. What are *a* and *b* in the function in part (A)? What do *a* and *b* mean in the context of the problem?**

In the function $f(x) = 75 \cdot 0.62^x$, $a = 75$, and $b = 0.62$.

The value of $a$ is the initial height of the ball, 75 in. The value of $b$ is the ratio between rebounds. Each rebound is 0.62, or 62%, of the height of the previous rebound.

For additional support when completing your homework, go to PearsonTEXAS.com.

**Identify the domain and range of the function. Express the range using an inequality.**

**1.** $f(x) = 15 \cdot 1.79^x$

**2.** $f(x) = 460 \cdot 0.17^x$

**3.** $f(x) = -3.5 \cdot 12^x$

**4.** $f(x) = -24 \cdot 0.999^x$

**Find $f(0)$, $f(4)$, and $f(-2)$ for each function. Round to the nearest tenth.**

**5.** $f(x) = 8 \cdot 2.02^x$

**6.** $f(x) = 107 \cdot 0.387^x$

**7.** $f(x) = -6.4 \cdot 1.49^x$

**8.** $f(x) = -5 \cdot 0.835^x$

**9. Select Tools to Solve Problems (1)(C)** The table at the right shows the length and weight of a species of fish.

Fish Lengths and Weights

| Length (in.) | 8 | 10 | 12 | 14 | 16 |
|---|---|---|---|---|---|
| Weight (lb) | 1.20 | 1.40 | 1.64 | 1.92 | 2.25 |

**a.** Select one of the following tools to use to find an exponential model for the data and predict the weight of a fish that is 24 in. long. Explain your choice.

paper and pencil     technology     manipulatives

**b.** Find an exponential model for the data. Predict the weight of a fish that is 24 in. long.

**10. Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** A culture is started with 50,000 bacteria, and then an antibiotic is added to the culture. The table at the right shows the number of bacteria after the antibiotic is added.

Bacteria Population

| Time Since Antibiotic Added (hr) | Number of Bacteria |
|---|---|
| 0 | 50,000 |
| 1 | 42,100 |
| 2 | 35,448 |
| 3 | 29,847 |
| 4 | 25,131 |

**a.** Find an exponential model that is a reasonable fit for the data.

**b.** How long will it take for the number of bacteria to fall below 5000?

**c.** What is the $y$-intercept of your function? What does it mean in the context of the problem?

**d.** What are the values of $a$ and $b$ for your function? What do these values mean in the context of the problem?

**e.** What are the real-world domain and range of the function? Express each using inequalities.

**f.** Graph the function.

**11. Select Tools to Solve Problems (1)(C)** The table below shows the balance in Mr. Harris's retirement savings account for several years after he retired.

| Retirement Account Balance | | | | | |
|---|---|---|---|---|---|
| Years Since Retirement | 0 | 1 | 2 | 3 | 4 |
| Balance ($) | 275,000 | 265,375 | 256,087 | 247,124 | 238,475 |

**a.** Use technology to find an exponential model that is a reasonable fit for the data.

**b.** Predict when the balance will be less than $100,000, assuming that the pattern continues.

**12. Use Representations to Communicate Mathematical Ideas (1)(E)** Samantha took out a loan that does not require her to make any payments for 12 months, although interest is being charged to her account. The table below shows the total amount she owes each month after taking the loan.

| Loan Balance | | | | | |
|---|---|---|---|---|---|
| Month | 0 | 1 | 2 | 3 | 4 |
| Total Amount Owed ($) | 3700 | 3752 | 3808 | 3858 | 3912 |

**a.** Find an exponential model that is a reasonable fit for the data.

**b.** How much will Samantha owe at the end of 12 months, assuming she does not make any payments before then?

**c.** What is the $y$-intercept of your function? What does it mean in the context of the problem?

**d.** What are the values of $a$ and $b$ for your function? What do these values mean in the context of the problem?

**e.** What are the real-world domain and range of the function? Express each using inequalities.

**f.** Graph the function.

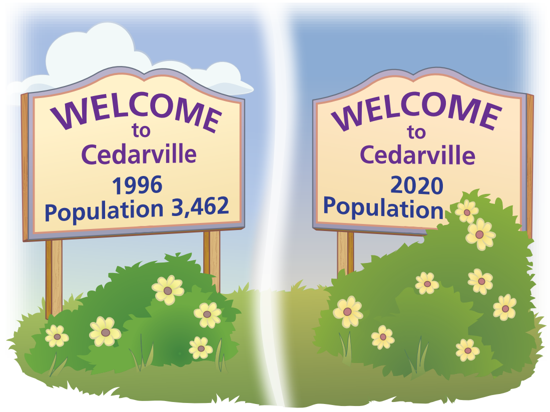
**13. Apply Mathematics (1)(A)** The function $f(x) = 0.973 \cdot 1.1872^x$ represents the value in millions of dollars of a software company, where $x$ is the number of years since the company was started.

**a.** What is the $y$-intercept of the function, and what does it mean in the context of the problem?

**b.** What are the $a$ and $b$ values of the function, and what do these values mean in the context of the problem?

**c.** According to the model, predict what the value of the company will be 10 years after it was started.

14. The function $f(x) = 3462 \cdot 0.925^x$ represents the population of a town, where $x$ is the number of years since 1996.




   a. What is the $y$-intercept of the function, and what does it mean in the context of the problem?

   b. What are the $a$ and $b$ values of the function, and what do these values mean in the context of the problem?

   c. According to the model, predict what the population will be in 2020.

**For $f(x) = a \cdot b^x$, determine whether each statement is *always*, *sometimes*, or *never true*.**

15. The $y$-intercept is $a$.

16. The value of $f(x)$ increases as the value of $x$ increases.

17. If $a > 0$ and $b > 0$, the graph of the function is entirely in Quadrant 1 of the coordinate plane.

18. The range includes negative numbers.

## TEXAS End-of-Course PRACTICE

19. What is the range of $f(x) = 43 \cdot 2.4^x$?

   A. $f(x) > 0$ B. $f(x) > 2.4$ C. $f(x) > 103.2$ D. $0 < f(x) < 43$

20. The function $f(x) = 24{,}950 \cdot 0.7932^x$ models the value of a car, where $x$ is the age of the car in years. How old will the car be when the value of the car becomes less than \$5000?

   F. 2 years G. 3 years H. 5 years J. 7 years

21. The value of Edward's savings account is modeled by the function $f(x) = 975 \cdot 1.028^x$, where $x$ is the number of years since the account was opened. In how many years will the account value be double the original amount?

   A. 20 B. 25 C. 30 D. 35

22. The table shows the number of contestants remaining after each round of a singing competition. What is an exponential model for the data? Use your model to predict how many rounds are required to select a single winner.

**Singing Competition**

| Round | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Number of Contestants Remaining | 150 | 90 | 54 | 32 | 19 |

# Topic 9 Review

## TOPIC VOCABULARY

- asymptote, *p. 386*
- compound interest, *p. 396*
- decay factor, *p. 396*
- exponential decay, *p. 396*
- exponential function, *p. 386*
- exponential growth, *p. 395*
- growth factor, *p. 395*

## Check Your Understanding

**Choose the correct term to complete each sentence.**

**1.** For a function $y = a \cdot b^x$, where $a > 0$ and $b > 1$, $b$ is the __?__.

**2.** For a function $y = a \cdot b^x$, where $a > 0$ and $0 < b < 1$, $b$ is the __?__.

**3.** The function $y = a \cdot b^x$ models __?__ for $a > 0$ and $b > 1$.

**4.** The function $y = a \cdot b^x$ models __?__ for $a > 0$ and $0 < b < 1$.

## 9-1 Exponential Functions

### Quick Review

An **exponential function** involves repeated multiplication of an initial amount $a$ by the same positive number $b$. The general form of an exponential function is $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

### Example

**What is the graph of $y = \frac{1}{2} \cdot 5^x$?**

Make a table of values. Graph the ordered pairs.

| $x$ | $y$ |
|---|---|
| −2 | $\frac{1}{50}$ |
| −1 | $\frac{1}{10}$ |
| 0 | $\frac{1}{2}$ |
| 1 | $\frac{5}{2}$ |
| 2 | $\frac{25}{2}$ |


### Exercises

**Evaluate each function for the domain $\{1, 2, 3\}$.**

**5.** $f(x) = 4^x$

**6.** $y = 0.01^x$

**7.** $y = 40\left(\frac{1}{2}\right)^x$

**8.** $f(x) = 3 \cdot 2^x$

**Graph each function.**

**9.** $f(x) = 2.5^x$

**10.** $y = 0.5(0.5)^x$

**11.** $f(x) = \frac{1}{2} \cdot 3^x$

**12.** $y = 0.1^x$

**13.** A population of 50 bacteria in a laboratory culture doubles every 30 min. The function $p(x) = 50 \cdot 2^x$ models the population, where $x$ is the number of 30-min periods.

**a.** How many bacteria will there be after 2 h?

**b.** How many bacteria will there be after 1 day?

## 9-2 Exponential Growth and Decay

### Quick Review

When $a > 0$ and $b > 1$, the function $y = a \cdot b^x$ models **exponential growth**. The base $b$ is called the **growth factor**. When $a > 0$ and $0 < b < 1$, the function $y = a \cdot b^x$ models **exponential decay**. In this case the base $b$ is called the **decay factor**.

### Example

**The population of a city is 25,000 and decreases 1% each year. Predict the population after 6 yr.**

| | |
|---|---|
| $y = 25{,}000 \cdot 0.99^x$ | Exponential decay function |
| $= 25{,}000 \cdot 0.99^6$ | Substitute 6 for $x$. |
| $\approx 23{,}537$ | Simplify. |

The population will be about 23,537 after 6 yr.

### Exercises

**Tell whether the function represents *exponential growth* or *exponential decay*. Identify the growth or decay factor.**

**14.** $y = 5.2 \cdot 3^x$

**15.** $f(x) = 7 \cdot 0.32^x$

**16.** $y = 0.15\left(\frac{3}{2}\right)^x$

**17.** $g(x) = 1.3\left(\frac{1}{4}\right)^x$

**18.** Suppose \$2000 is deposited in an account paying 2.5% interest compounded quarterly. What will the account balance be after 12 yr?

**19.** A band performs a free concert in a local park. There are 200 people in the crowd at the start of the concert. The number of people in the crowd grows 15% every half hour. How many people are in the crowd after 3 h? Round to the nearest person.

## 9-3 Modeling Exponential Data

### Quick Review

For some data, an exponential model is a good fit. You can enter the data into your calculator and use the regression feature to find an exponential model.

You can then use your model to answer questions about the data by examining the graphs or tables, or by using the function.

Exponential functions are written in the form $f(x) = a \cdot b^x$.

### Example

**The table shows the value of a computer. Find an exponential model, and find the computer's value after 7 years.**

| Computer Value | | | | | |
|---|---|---|---|---|---|
| Age | 0 | 1 | 2 | 3 | 4 |
| Value (\$) | 2200 | 1382 | 868 | 545 | 342 |

The data appear to be exponential. The calculator gives a regression model of $f(x) = 2200 \cdot 0.628^x$.

Use the table and the exponential model to see that the computer will be worth about \$85 after 7 years.

### Exercises

The table shows the population of geese in a park beginning in the year 2003. Use the table to answer Exercises 20–25.

| Geese Population | | | | | |
|---|---|---|---|---|---|
| Year | 0 | 1 | 2 | 3 | 4 |
| Population | 58 | 61 | 65 | 69 | 73 |

**20.** Find an exponential model to best fit the data.

**21.** Predict the population in the year 2015.

**22.** According to the model, in what year will the population first be greater than 150 geese?

**23.** What is the $y$-intercept of your function, and what does it mean in the context of the problem?

**24.** What are the $a$ and $b$ values of your function, and what do they mean in the context of the problem?

**25.** What are the real-world domain and range of the function?

## Multiple Choice

**Read each question. Then write the letter of the correct answer on your paper.**

1. If the graph of the function $y = x^2 - 6$ were shifted 3 units down, which equation could represent the shifted graph?

   **A.** $y = 3x^2 - 6$  **C.** $y = x^2 - 9$

   **B.** $y = x^2 - 3$  **D.** $y = 3x^2 - 3$

2. Which ordered pair is a solution of $3x - y < 20$?

   **F.** (7, 1)  **H.** (8, 0)

   **G.** (5, −6)  **J.** (−1, −4)

3. Brianna has a cylindrical glass that is 15 cm tall. The diameter of the base is 5 cm. About how much water can the glass hold?

   **A.** 75 $cm^3$  **C.** 295 $cm^3$

   **B.** 118 $cm^3$  **D.** 1178 $cm^3$

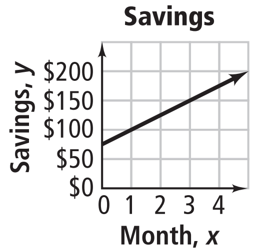
4. What is the solution of this system of equations?

$$\begin{aligned} -3x + y &= -2 \\ x + y &= -6 \end{aligned}$$

   **F.** (−2, −6)  **H.** (1, 5)

   **G.** (−1, −5)  **J.** (−3, −2)

5. Jeremiah made the graph at the right to show how much money he saved after working for a few months. Which of the following represents the amount of money Jeremiah had when he started working?




   **A.** $x$-intercept

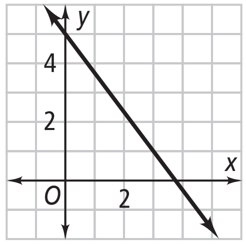
   **B.** $y$-intercept

   **C.** slope

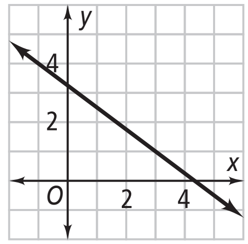
   **D.** domain

6. Eduardo is drawing the graph of a function. Each time the $x$-value increases by 3, the $y$-value decreases by 4. The function includes the point (1, 3). Which could be Eduardo's graph?

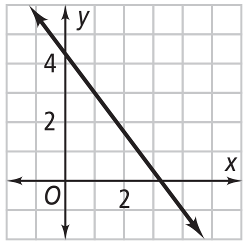
**F.**



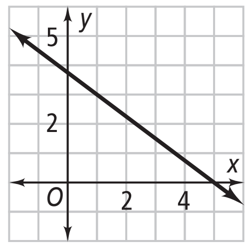

**H.**




**G.**




**J.**




7. The data shown in the table at the right represent points on a line. What is the $y$-intercept of the line?

| $x$ | $y$ |
|---|---|
| 2 | −1 |
| 3 | 1 |
| 4 | 3 |
| 5 | 5 |

   **A.** −5

   **B.** −3

   **C.** 0

   **D.** 2.5

8. What is the factored form of $3x^2 + 2xy - 8y^2$?

   **F.** $(x + y)(3x - 8y)$

   **G.** $(x + 4y)(3x - 2y)$

   **H.** $(x + 2y)(3x - 4y)$

   **J.** $(3x + 2y)(x - 4y)$

9. The formula for the area $A$ of a circle is $A = \pi r^2$, where $r$ is the radius of the circle. Which equation can be used to find the radius?

   **A.** $r = \frac{\sqrt{A\pi}}{\pi}$  **C.** $r = \frac{A^2}{\pi}$

   **B.** $r = \frac{A}{\pi}$  **D.** $r = \sqrt{A\pi}$

10. Which function has $y$-values that always increase when the corresponding $x$-values increase?

**F.** $y = |x| + 2$

**G.** $y = x^2 + 2$

**H.** $y = x + 2$

**J.** $y = -x - 1$

11. What is the solution of this system of equations?

$$x + 2y = 23$$
$$4x - y = -7$$

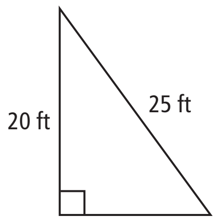
**A.** $(1, 11)$ **C.** $(-1, -11)$

**B.** $(-11, 1)$ **D.** $(11, 1)$

## Gridded Response

12. The formula $h = -16t^2 + c$ can be used to find the height $h$, in feet, of a falling object $t$ seconds after it is dropped from a height of $c$ feet. Suppose an object falls from a height of 40 ft. How long, in seconds, will the object take to reach the ground? Round your answer to the nearest tenth.

13. A 25-foot ladder rests against a wall. The top of the ladder reaches 20 feet high on the wall. How far away from the wall is the bottom of the ladder in feet?




14. What is the fifth term in the sequence below?

3.25, 4, 4.75, 5.5, . . .

15. What is the solution of the following proportion?

$$\frac{-a}{4} = \frac{-3(a - 2)}{6}$$

16. Mariah made a model of a square pyramid. The height $h$ of the pyramid is 6 in. The area of the base $B$ is 36 in.$^2$. What is the volume $V$, in cubic inches, of the pyramid? Use the formula $V = \frac{1}{3}Bh$.

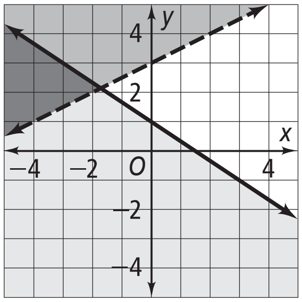
## Constructed Response

17. The list below shows the heights, in inches, of the students in Corey's class.

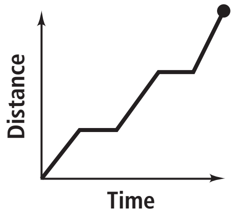
60, 64, 58, 57, 60, 65, 51, 53, 57, 56

How many students are more than 5 ft tall?

18. Write the system of inequalities for the graph below. Show your work.

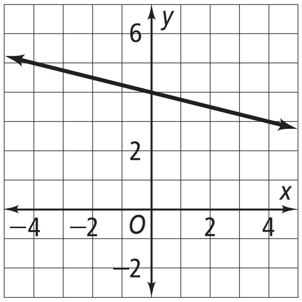



19. Mr. Wong drove to the grocery store. The graph at the right shows his distance from home during the drive. How many times did Mr. Wong stop the car before reaching the grocery store?




20. The volume of a rectangular prism is 720 in.$^3$. The height of the prism is 10 in. The width is 4 in. What is the length, in inches?

21. What is the slope of the line below?




22. Your cellphone plan costs \$39.99 per month plus \$.10 for every text message that you receive or send. This month, you receive 7 text messages and send 10 text messages. What is your bill, in dollars, for this month?

# Topic 9

## Lesson 9-1

**Evaluate each function over the domain {−1, 0, 1, 2}. As the values of the domain increase, do the values of the function *increase* or *decrease*?**

**1.** $y = 3^x$
**2.** $y = \left(\frac{3}{4}\right)^x$
**3.** $y = 1.5^x$
**4.** $y = \left(\frac{1}{2}\right) \cdot 3^x$
**5.** $y = -3 \cdot 7^x$
**6.** $y = -(4)^x$
**7.** $y = 3 \cdot \left(\frac{1}{5}\right)^x$
**8.** $y = 2^x$
**9.** $y = 2 \cdot 3^x$
**10.** $y = (0.8)^x$
**11.** $y = 2.5^x$
**12.** $y = -4 \cdot (0.2)^x$

**Graph each exponential function. Identify the domain, range, *y*-intercept, and asymptote.**

**13.** $y = \frac{1}{2} \cdot 3^x$
**14.** $y = -4 \cdot 2^x$
**15.** $y = 4 \cdot (0.3)^x$

**Write and solve an exponential equation to answer each question.**

**16.** Suppose an investment of \$5,000 doubles every 12 years. How much is the investment worth after 36 years? After 48 years?

**17.** Suppose 15 animals are taken to an island, and then their population triples every 8 months. How many animals will there be in 4 years?

**18.** The population of a city this year is 34,500. The population is expected to grow by 3% each year. What will the population of the city be in 12 years?

## Lesson 9-2

**Identify each function as *exponential growth* or *exponential decay*. Then identify the growth factor or decay factor.**

**19.** $y = 8^x$
**20.** $y = \frac{3}{4}2^x$
**21.** $y = 9\left(\frac{1}{2}\right)^x$
**22.** $y = 4 \cdot 9^x$
**23.** $y = 0.65^x$
**24.** $y = 3 \cdot 1.5^x$
**25.** $y = \frac{2}{5}\left(\frac{1}{4}\right)^x$
**26.** $y = 0.1 \cdot 0.9^x$
**27.** $y = 0.7 \cdot 3.3^x$

**Graph each exponential function, and identify the *y*-intercept and the asymptote.**

**28.** $y = \frac{1}{2}\left(\frac{1}{4}\right)^x$
**29.** $y = 4 \cdot (0.2)^x$
**30.** $y = 3.5^x$

## Lesson 9-2 *continued*

**Write an exponential function that describes each situation.**

**31.** The value of a function decreases by a factor of 0.6 with every unit increase in $x$. When $x = 0$, the value of the function is 10.

**32.** The value of a function increases by a factor of 8.5%. When $x = 0$, the value of the function is 22.

**Write an exponential function that describes each situation. Find the balance in each account after the given period.**

**33.** $200 principal earning 4% compounded annually, after 5 years

**34.** $1000 principal earning 3.6% compounded monthly, after 10 years

**35.** $3000 investment losing 8% compounded annually, after 3 years

**Find the balance in each account.**

**36.** You deposit $2500 in a savings account with 3% interest compounded annually. What is the balance in the account after 6 years?

**37.** You deposit $750 in an account with 7% interest compounded semiannually. What is the balance in the account after 4 years?

**38.** You deposit $520 in an account with 4% interest compounded monthly. What is the balance in the account after 5 years?

## Lesson 9-3

**39.** The table shows the population (in thousands) of the country of Algeria from 1950 to 1990, where $x$ is the number of years since 1950.

| Year Since 1950 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|---|
| Population (thousands) | 8893 | 9842 | 10,909 | 11,963 | 12,932 | 16,140 | 18,806 | 22,008 | 25,190 |

SOURCE: ***United States Census Bureau***

**a.** Find an exponential model that is a reasonable fit for the data.

**b.** What is the predicted population of Algeria in 2020, assuming that this trend continues?

**40.** Camilla had a collection of 140 coins from all over the world. She decided to give some coins away to friends. The table below shows the number of coins remaining in her collection.

| Day | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Number of Coins | 140 | 102 | 76 | 55 | 33 |

**a.** Find an exponential model that is a reasonable fit for the data.

**b.** How many coins will Camilla have left after 7 days, assuming that her pattern continues?

# Reference

## Table 1 | Measures

| | United States Customary | Metric |
|---|---|---|
| **Length** | 12 inches (in.) = 1 foot (ft)<br>36 in. = 1 yard (yd)<br>3 ft = 1 yard<br>5280 ft = 1 mile (mi)<br>1760 yd = 1 mile | 10 millimeters (mm) = 1 centimeter (cm)<br>100 cm = 1 meter (m)<br>1000 mm = 1 meter<br>1000 m = 1 kilometer (km) |
| **Area** | 144 square inches (in.$^2$) = 1 square foot (ft$^2$)<br>9 ft$^2$ = 1 square yard (yd$^2$)<br>43,560 ft$^2$ = 1 acre (a)<br>4840 yd$^2$ = 1 acre | 100 square millimeters (mm$^2$) = 1 square centimeter (cm$^2$)<br>10,000 cm$^2$ = 1 square meter (m$^2$)<br>10,000 m$^2$ = 1 hectare (ha) |
| **Volume** | 1728 cubic inches (in.$^3$) = 1 cubic foot (ft$^3$)<br>27 ft$^3$ = 1 cubic yard (yd$^3$) | 1000 cubic millimeters (mm$^3$) = 1 cubic centimeter (cm$^3$)<br>1,000,000 cm$^3$ = 1 cubic meter (m$^3$) |
| **Liquid Capacity** | 8 fluid ounces (fl oz) = 1 cup (c)<br>2 c = 1 pint (pt)<br>2 pt = 1 quart (qt)<br>4 qt = 1 gallon (gal) | 1000 milliliters (mL) = 1 liter (L)<br>1000 L = 1 kiloliter (kL) |
| **Weight or Mass** | 16 ounces (oz) = 1 pound (lb)<br>2000 pounds = 1 ton (t) | 1000 milligrams (mg) = 1 gram (g)<br>1000 g = 1 kilogram (kg)<br>1000 kg = 1 metric ton |
| **Temperature** | 32°F = freezing point of water<br>98.6°F = normal human body temperature<br>212°F = boiling point of water | 0°C = freezing point of water<br>37°C = normal human body temperature<br>100°C = boiling point of water |

| | Customary Units and Metric Units |
|---|---|
| **Length** | 1 in. = 2.54 cm<br>1 mi ≈ 1.61 km<br>1 ft ≈ 0.305 m |
| **Capacity** | 1 qt ≈ 0.946 L |
| **Weight and Mass** | 1 oz ≈ 28.4 g<br>1 lb ≈ 0.454 kg |

| Time | | |
|---|---|---|
| 60 seconds (s) = 1 minute (min) | 4 weeks (approx.) = 1 month (mo) | 12 months = 1 year |
| 60 minutes = 1 hour (h) | 365 days = 1 year (yr) | 10 years = 1 decade |
| 24 hours = 1 day (d) | 52 weeks (approx.) = 1 year | 100 years = 1 century |
| 7 days = 1 week (wk) | | |

# Table 2 | Reading Math Symbols

| Symbols | Words |
|---|---|
| $\cdot$ | multiplication sign, times ($\times$) |
| $=$ | equals |
| $\stackrel{?}{=}$ | Are the statements equal? |
| $\approx$ | is approximately equal to |
| $\neq$ | is not equal to |
| $<$ | is less than |
| $>$ | is greater than |
| $\leq$ | is less than or equal to |
| $\geq$ | is greater than or equal to |
| $\cong$ | is congruent to |
| $\pm$ | plus or minus |
| ( ) | parentheses for grouping |
| [ ] | brackets for grouping |
| { } | set braces |
| % | percent |
| $\lvert a \rvert$ | absolute value of *a* |
| . . . | and so on |
| $-a$ | opposite of *a* |
| $\pi$ | pi, an irrational number, approximately equal to 3.14 |
| $^\circ$ | degree(s) |
| $a^n$ | *n*th power of a |
| $\sqrt{x}$ | nonnegative square root of *x* |

| Symbols | Words |
|---|---|
| $\frac{1}{a}, a \neq 0$ | reciprocal of *a* |
| $a^{-n}$ | $\frac{1}{a^n}, a \neq 0$ |
| $\overleftrightarrow{AB}$ | line through points *A* and *B* |
| $\overline{AB}$ | segment with endpoints *A* and *B* |
| $AB$ | length of $\overline{AB}$; distance between points *A* and *B* |
| $\angle A$ | angle *A* |
| $m\angle A$ | measure of angle *A* |
| $\triangle ABC$ | triangle *ABC* |
| $(x, y)$ | ordered pair |
| $x_1, x_2, \ldots$ | specific values of the variable *x* |
| $y_1, y_2, \ldots$ | specific values of the variable *y* |
| $f(x)$ | *f* of *x*; the function value at *x* |
| $m$ | slope of a line |
| $b$ | *y*-intercept of a line |
| $a:b$ | ratio of *a* to *b* |
| ^ | raised to a power (in a spreadsheet formula) |
| * | multiply (in a spreadsheet formula) |
| / | divide (in a spreadsheet formula) |

Reference

# Properties and Formulas

### Order of Operations
**1.** Perform an operation(s) inside grouping symbols.
**2.** Simplify powers.
**3.** Multiply and divide from left to right.
**4.** Add and subtract from left to right.

### Commutative Property of Addition
For every real number $a$ and $b$, $a + b = b + a$.

### Commutative Property of Multiplication
For every real number $a$ and $b$, $a \cdot b = b \cdot a$.

### Associative Property of Addition
For every real number $a$, $b$, and $c$,
$(a + b) + c = a + (b + c)$.

### Associative Property of Multiplication
For every real number $a$, $b$, and $c$,
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

### Identity Property of Addition
For every real number $a$, $a + 0 = a$.

### Identity Property of Multiplication
For every real number $a$, $1 \cdot a = a$.

### Multiplication Property of $-1$
For every real number $a$, $-1 \cdot a = -a$.

### Zero Property of Multiplication
For every real number $a$, $a \cdot 0 = 0$.

### Inverse Property of Addition
For every real number $a$, there is an additive inverse $-a$ such that $a + (-a) = 0$.

### Inverse Property of Multiplication
For every nonzero number $a$, there is a multiplicative inverse such that $a \cdot \frac{1}{a} = 1$.

### Distributive Property
For every real number $a$, $b$, and $c$:
$a(b + c) = ab + ac$
$(b + c)a = ba + ca$
$a(b - c) = ab - ac$
$(b - c)a = ba - ca$

### Addition Property of Equality
For every real number $a$, $b$, and $c$, if $a = b$, then $a + c = b + c$.

### Subtraction Property of Equality
For every real number $a$, $b$, and $c$, if $a = b$, then $a - c = b - c$.

### Multiplication Property of Equality
For every real number $a$, $b$, and $c$, if $a = b$, then $a \cdot c = b \cdot c$.

### Division Property of Equality
For every real number $a$, $b$, and $c$, where $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

### Percent Proportion
$\frac{a}{b} = \frac{p}{100}$, where $b \neq 0$.

### Percent Equation
$a = p\% \cdot b$, where $b \neq 0$.

### Simple Interest Formula
$I = prt$

### Percent of Change
$p\% = \frac{\text{amount of increase or decrease}}{\text{original amount}}$
amount of increase = new amount − original amount
amount of decrease = original amount − new amount

### Relative Error
$\text{relative error} = \frac{|\text{measured or estimated value} - \text{actual value}|}{\text{actual value}}$

*The following properties of inequality are also true for $\geq$ and $\leq$.*

### Addition Property of Inequality
For every real number $a$, $b$, and $c$,
if $a > b$, then $a + c > b + c$;
if $a < b$, then $a + c < b + c$.

### Subtraction Property of Inequality
For every real number $a$, $b$, and $c$,
if $a > b$, then $a - c > b - c$;
if $a < b$, then $a - c < b - c$.

### Multiplication Property of Inequality
For every real number $a$, $b$, and $c$, where $c > 0$,
if $a > b$, then $ac > bc$;
if $a < b$, then $ac < bc$.
For every real number $a$, $b$, and $c$, where $c < 0$,
if $a > b$, then $ac < bc$;
if $a < b$, then $ac > bc$.

### Division Property of Inequality
For every real number $a$, $b$, and $c$, where $c > 0$,
if $a > b$, then $\frac{a}{c} > \frac{b}{c}$;
if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.
For every real number $a$, $b$, and $c$, where $c < 0$,
if $a > b$, then $\frac{a}{c} < \frac{b}{c}$;
if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

### Reflexive Property of Equality
For every real number $a$, $a = a$.

### Symmetric Property of Equality
For every real number $a$ and $b$,
if $a = b$, then $b = a$.

### Transitive Property of Equality
For every real number $a$, $b$, and $c$,
if $a = b$ and $b = c$, then $a = c$.

### Transitive Property of Inequality
For every real number $a$, $b$, and $c$,
if $a < b$ and $b < c$, then $a < c$.

## Topic 1 Solving Equations and Inequalities

### Distance Traveled
$d = rt$

### Temperature
$C = \frac{5}{9}(F - 32)$

### Cross Products of a Proportion
If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

## Topic 2 An Introduction to Functions

### Function Notation
Functions are represented as equations involving $x$ and $y$, such as $y = 2x + 3$. The same equation can be written in function notation $f(x) = 2x + 3$.

Notice $f(x)$ replaces $y$ and is read "$f$ of $x$". The letter $f$ is the name of the function, not a variable.

## Topic 3 Linear Functions

### Slope
$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$

### Direct Variation
A direct variation is a relationship that can be represented by a function of the form $y = kx$, where $k \neq 0$.

### Slope-Intercept Form of a Linear Equation
The slope-intercept form of a linear equation is $y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept.

### Point-Slope Form of a Linear Equation
The point-slope form of the equation of a nonvertical line that passes through the point $(x_1, y_1)$ with slope $m$ is $y - y_1 = m(x - x_1)$.

### Standard Form of a Linear Equation
The standard form of a linear equation is $Ax + By = C$, where $A$, $B$, and $C$ are real numbers and $A$ and $B$ are not both zero.

### Slopes of Parallel Lines
Nonvertical lines are parallel if they have the same slope and different $y$-intercepts. Any two vertical lines are parallel.

### Slopes of Perpendicular Lines
Two lines are perpendicular if the product of their slopes is $-1$. A vertical line and horizontal line are perpendicular.

### Function Family: Linear

| | |
|---|---|
| Parent | $y = f(x)$ |
| Translate | |
| horizontal by $\lvert c \rvert$ | $y = f(x - c)$ |
| vertical by $\lvert d \rvert$ | $y = f(x) + d$ |
| Reflection | |
| across the $x$-axis | $y = -f(x)$ |
| across the $y$-axis | $y = f(-x)$ |
| Slope Change | |
| $a > 1$, graph steeper<br>$0 < a < 1$, graph less steep | $y = af(x)$ |
| $b > 1$, graph steeper<br>$0 < b < 1$, graph less steep | $y = f(bx)$ |

## Topic 4 Systems of Equations and Inequalities

### Solutions of Systems of Linear Equations
A system of linear equations can have one solution, no solution, or infinitely many solutions:
- If the lines have different slopes, the lines intersect, so there is one solution.
- If the lines have the same slopes and different $y$-intercepts, the lines are parallel, so there are no solutions.
- If the lines have the same slopes and the same $y$-intercepts, the lines are the same, so there are infinitely many solutions.

## Topic 5 Exponents and Radicals

### Zero as an Exponent
For every nonzero number $a$, $a^0 = 1$.

## Negative Exponent

For every nonzero number $a$ and rational number $n$, $a^{-n} = \frac{1}{a^n}$.

## Multiplying Powers With the Same Base

For every nonzero number $a$ and rational numbers $m$ and $n$, $a^m \cdot a^n = a^{m+n}$.

## Dividing Powers With the Same Base

For every nonzero number $a$ and rational numbers $m$ and $n$, $\frac{a^m}{a^n} = a^{m-n}$.

## Raising a Power to a Power

For every nonzero number $a$ and rational numbers $m$ and $n$, $(a^m)^n = a^{mn}$.

## Raising a Product to a Power

For every nonzero number $a$ and $b$ and rational number $n$, $(ab)^n = a^n b^n$.

## Raising a Quotient to a Power

For every nonzero number $a$ and $b$ and rational number $n$, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

## Properties of Rational Exponents

If the $n$th root of $a$ is a real number and $m$ is an integer, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$. If $m$ is negative, $a \neq 0$.

## Multiplication Property of Square Roots

For every number $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

## Division Property of Square Roots

For every number $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

## The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse: $a^2 + b^2 = c^2$.

## The Converse of the Pythagorean Theorem

If a triangle has sides of lengths $a$, $b$, and $c$, and $a^2 + b^2 = c^2$, then the triangle is a right triangle with hypotenuse of length $c$.

# Topic 6 Sequences

## Arithmetic Sequence

The explicit formula for an arithmetic sequence is $A(n) = A(1) + (n - 1)d$, where $A(n)$ is the $n$th term, $A(1)$ is the first term, $n$ is the term number, and $d$ is the common difference.

The recursive definition for an arithmetic sequence with common difference $d$ has two parts:

| | |
|---|---|
| $A(1) =$ first term | Initial condition, or starting value |
| $A(n) = A(n - 1) + d$, for $n \geq 2$ | Recursive formula |

## Geometric Sequence

The explicit formula for a geometric sequence is $A(n) = A(1) \cdot r^{n-1}$, where $A(n)$ is the $n$th term, $A(1)$ is the first term, $n$ is the term number, and $r$ is the common ratio.

The recursive definition for a geometric sequence with common ratio $r$ has two parts:

| | |
|---|---|
| $A(1) = a$ | Initial condition, or starting value |
| $A(n) = A(n - 1) \cdot r$, for $n \geq 2$ | Recursive formula |

# Topic 7 Polynomials and Factoring

## Factoring Special Cases

For every nonzero number $a$ and $b$:

$a^2 - b^2 = (a + b)(a - b)$

$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$

# Topic 8 Quadratic Functions and Equations

## Graph of a Quadratic Function

The graph of $y = ax^2 + bx + c$, where $a \neq 0$, has the line $x = \frac{-b}{2a}$ as its axis of symmetry. The $x$-coordinate of the vertex is $\frac{-b}{2a}$.

## Function Family: Quadratic

| | |
|---|---|
| Parent | $y = x^2$ |
| Translation | |
| horizontal | $y = (x - c)^2$ |
| $c > 0$, $c$ units right | |
| $c < 0$, $\lvert c \rvert$ units left | |
| vertical | $y = x^2 + d$ |
| $d > 0$, $d$ units up | |
| $d < 0$, $\lvert d \rvert$ units down | |
| Stretch, Compression, and Reflection | |
| horizontal | $y = (bx)^2$ |
| compression ($\lvert b \rvert > 1$) | |
| stretch ($0 < \lvert b \rvert < 1$) | |
| reflection across the $y$-axis ($b < 0$) | |
| vertical | $y = a(x)^2$ |
| stretch ($\lvert a \rvert > 1$) | |
| compression ($0 < \lvert a \rvert < 1$) | |
| reflection across the $x$-axis ($a < 0$) | |

## Vertex Form of a Quadratic Function

The quadratic function $f(x) = a(x - h)^2 + k$ is written in vertex form.

Axis of symmetry: $x = h$

Vertex: $(h, k)$

Maximum or minimum value: $k$

## Zero-Product Propert

For every real number $a$ and $b$, if $ab = 0$, then $a = 0$ or $b = 0$.

## Completing the Square

By adding $\left(\frac{b}{2}\right)^2$ to the expression $x^2 + bx$, it forms a perfect-square trinomial. $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

## Quadratic Formula

If $ax^2 + bx + c = 0$, where $a \neq 0$, then

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

## Property of the Discriminant

For the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, the value of the discriminant $b^2 - 4ac$ tells you the number of solutions.

- If $b^2 - 4ac > 0$, there are two real solutions.
- If $b^2 - 4ac = 0$, there is one real solution.
- If $b^2 - 4ac < 0$, there are no real solutions.

# Topic 9 Exponential Functions and Equations

## Exponential Function

An exponential function is a function of the form $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, $b \neq 1$, and $x$ is a real number. The horizontal asymptote for an exponential function of the form $y = a \cdot b^x$ is the $x$-axis $(y = 0)$.

## Compound Interest Formula

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

## Exponential Growth and Decay

An exponential function has the form $y = a \cdot b^x$, where $a$ is a nonzero constant, $b$ is greater than 0 and not equal to 1, and $x$ is a real number.

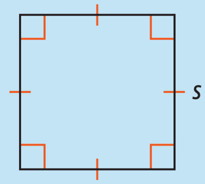
- The function $y = a \cdot b^x$, where $b$ is the growth factor, models exponential growth for $a > 0$ and $b > 1$.
- The function $y = a \cdot b^x$, where $b$ is the decay factor, models exponential decay for $a > 0$ and $0 < b < 1$.

# Formulas of **Geometry**

You will use a number of geometric formulas as you work through your algebra book. Here are some perimeter, area, and volume formulas.

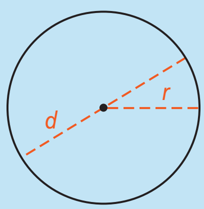

$P = 2\ell + 2w$
$A = \ell w$

**Rectangle**



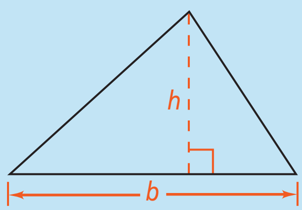

$P = 4s$
$A = s^2$

**Square**



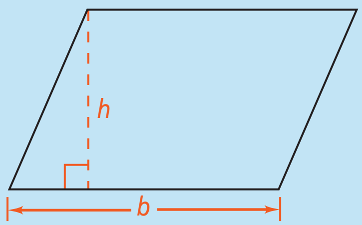

$C = 2\pi r$ or $C = \pi d$
$A = \pi r^2$

**Circle**



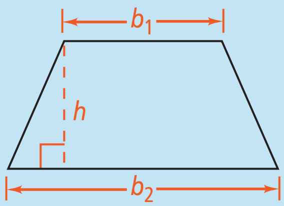

$A = \frac{1}{2}bh$

**Triangle**



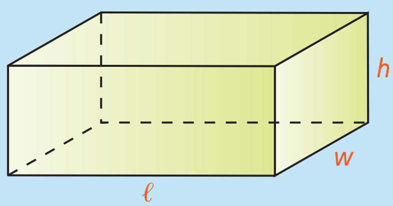

$A = bh$

**Parallelogram**



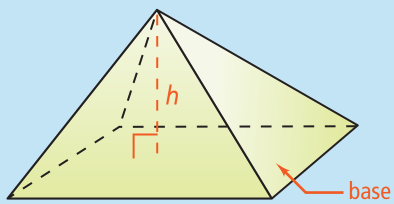

$A = \frac{1}{2}(b_1 + b_2)h$

**Trapezoid**



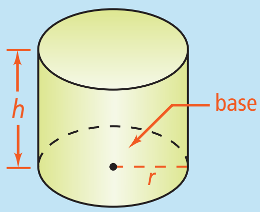

$V = Bh$
$V = \ell wh$

**Right Prism**



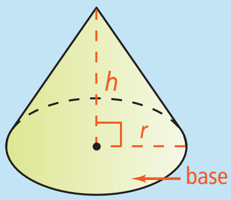

$V = \frac{1}{3}Bh$

**Pyramid**



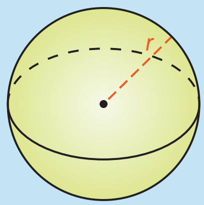

$V = Bh$
$V = \pi r^2 h$

**Right Cylinder**




$V = \frac{1}{3}Bh$
$V = \frac{1}{3}\pi r^2 h$

**Right Cone**




$V = \frac{4}{3}\pi r^3$

**Sphere**

# Visual Glossary

## English | Spanish

### A

**Arithmetic sequence (p. 246)** A number sequence formed by adding a fixed number to each previous term to find the next term. The fixed number is called the common difference.

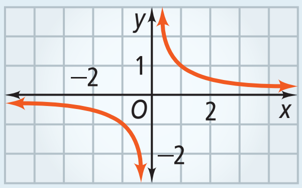
**Progresión aritmética (p. 246)** En una progresión aritmética la diferencia entre términos consecutivos es un número constante. El número constante se llama la diferencia común.

**Example** 4, 7, 10, 13, . . . is an arithmetic sequence.

**Asymptote (p. 384)** A line that the graph of a function gets closer to as $x$ or $y$ gets larger in absolute value.

**Asíntota (p. 384)** Línea recta a la que la gráfica de una función se acerca indefinidamente, mientras el valor absoluto de $x$ o $y$ aumenta.

**Example**

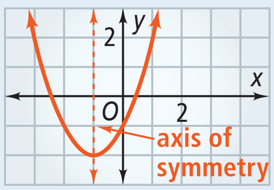



The $y$-axis is a vertical asymptote for $y = \frac{1}{x}$. The $x$-axis is a horizontal asymptote for $y = \frac{1}{x}$.

**Axis of symmetry (p. 326)** The line that divides a parabola into two matching halves.

**Eje de simetría (p. 326)** El eje de simetría es la línea que divide una parábola en dos mitades exactamente iguales.

**Example**




### B

**Binomial (p. 268)** A polynomial of two terms.

**Binomio (p. 268)** Polinomio compuesto de dos términos.

**Example** $3x + 7$ is a binomial.

### C

**Causation (p. 144)** When a change in one quantity causes a change in a second quantity. A correlation between quantities does not always imply causation.

**Causalidad (p. 144)** Cuando un cambio en una cantidad causa un cambio en una segunda cantidad. Una correlación entre las cantidades no implica siempre la causalidad.

**Common difference (p. 246)** The difference between consecutive terms of an arithmetic sequence.

**Diferencia común (p. 246)** La diferencia común es la diferencia entre los términos consecutivos de una progresión aritmética.

**Example** The common difference is 3 in the arithmetic sequence 4, 7, 10, 13, . . .

| English | Spanish |
|---|---|

**Common ratio (p. 246)** The fixed number used to find terms in a geometric sequence.

**Razón común (p. 246)** Número constante que se usa para hallar los términos en una progresión geométrica.

**Example** The common ratio is $\frac{1}{3}$ in the geometric sequence $9, 3, 1, \frac{1}{3}, \ldots$

**Completing the square (p. 367)** A method of solving quadratic equations. Completing the square turns every quadratic equation into the form $x^2 = c$.

**Completar el cuadrado (p. 367)** Método para solucionar ecuaciones cuadráticas. Cuando se completa el cuadrado, se transforma la ecuación cuadrática a la fórmula $x^2 = c$.

**Example** $x^2 + 6x - 7 = 9$ is rewritten as $(x + 3)^2 = 25$ by completing the square.

**Compound inequalities (p. 40)** Two inequalities that are joined by *and* or *or*.

**Desigualdades compuestas (p. 40)** Dos desigualdades que están enlazadas por medio de una *y* o una *o*.

**Examples** $5 < x$ and $x < 10$
$14 < x$ or $x \leq -3$

**Compound interest (p. 395)** Interest paid on both the principal and the interest that has already been paid.

**Interés compuesto (p. 395)** Interés calculado tanto sobre el capital como sobre los intereses ya pagados.

**Example** For an initial deposit of \$1000 at a 6% interest rate with interest compounded quarterly, the function $y = 1000\left(\frac{0.06}{4}\right)^x$ gives the account balance $y$ after $x$ years.

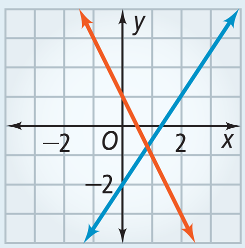
**Compression (p. 340)** A compression is a transformation that decreases the distance between corresponding points of a graph.

**Compresión (p. 340)** Una compresión es una transformación que disminuye la distancia entre puntos que se corresponden en una gráfica.

**Consistent system (p. 160)** A system of equations that has at least one solution is consistent.

**Sistema consistente (p. 160)** Un sistema de ecuaciones que tiene por lo menos una solución es consistente.

**Example**




**Constant of variation for direct variation (p. 108)** The nonzero constant $k$ in the function $y = kx$.

**Constante de variación en variaciones directas (p. 108)** La constante $k$ cuyo valor no es cero en la función $y = kx$.

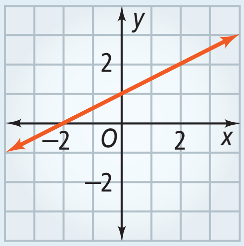
**Example** For the direct variation $y = 24x$, 24 is the constant of variation.

## English | Spanish

**Continuous graph (p. 70)** A graph that is unbroken.

**Gráfica continua (p. 70)** Una gráfica continua es una gráfica ininterrumpida.

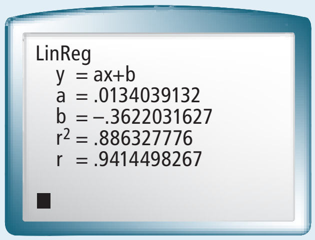
**Example**




**Correlation coefficient (p. 144)** A number from −1 to 1 that tells you how closely the equation of the line of best fit models the data.

**Coeficiente de correlación (p. 144)** Número de −1 a 1 que indica con cuánta exactitud la línea de mejor encaje representa los datos.

**Example**




The correlation coefficient is approximately 0.94.

**Cross products (of a proportion) (p. 22)** In a proportion $\frac{a}{b} = \frac{c}{d}$, the products *ad* and *bc*. These products are equal.

**Productos cruzados (de una proporción) (p. 22)** En una proporción $\frac{a}{b} = \frac{c}{d}$, los productos *ad* y *bc*. Estos productos son iguales.

**Example** The cross products for $\frac{3}{4} = \frac{6}{8}$ are $3 \cdot 8$ and $4 \cdot 6$.

## D

**Decay factor (p. 395)** 1 minus the percent rate of change, expressed as a decimal, for an exponential decay situation.

**Factor de decremento (p. 395)** 1 menos la tasa porcentual de cambio, expresada como decimal, en una situación de reducción exponencial.

**Example** The decay factor of the function $y = 5(0.3)^x$ is 0.3.

**Degree of a monomial (p. 268)** The sum of the exponents of the variables of a monomial.

**Grado de un monomio (p. 268)** La suma de los exponentes de las variables de un monomio.

**Example** $-4x^3y^2$ is a monomial of degree 5.

| English | Spanish |
|---|---|
| **Degree of a polynomial (p. 268)** The highest degree of any term of the polynomial. | **Grado de un polinomio (p. 268)** El grado de un polinomio es el grado mayor de cualquier término del polinomio. |

**Example** The polynomial $P(x) = x^6 + 2x^3 - 3$ has degree 6.

| | |
|---|---|
| **Dependent system (p. 160)** A system of equations that does not have a unique solution. | **Sistema dependiente (p. 160)** Sistema de ecuaciones que no tiene una solución única. |

**Example** The system $\begin{cases} y = 2x + 3 \\ -4x + 2y = 6 \end{cases}$ represents two equations for the same line, so it has many solutions. It is a dependent system.

| | |
|---|---|
| **Dependent variable (p. 59)** A variable that provides the output values of a function. | **Variable dependiente (p. 59)** Variable de la que dependen los valores de salida de una función. |

**Example** In the equation $y = 3x$, $y$ is the dependent variable.

| | |
|---|---|
| **Difference of two squares (p. 296)** A difference of two squares is an expression of the form $a^2 - b^2$. It can be factored as $(a + b)(a - b)$. | **Diferencia de dos cuadrados (p. 296)** La diferencia de dos cuadrados es una expresión de la forma $a^2 - b^2$. Se puede factorizar como $(a + b)(a - b)$. |

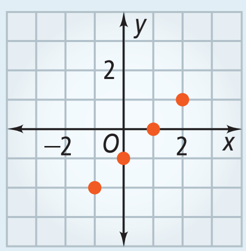
**Examples** $25a^2 - 4 = (5a + 2)(5a - 2)$
$m^6 - 1 = (m^3 + 1)(m^3 - 1)$

| | |
|---|---|
| **Direct variation (p. 108)** A linear function defined by an equation of the form $y = kx$, where $k \neq 0$. | **Variación directa (p. 108)** Una función lineal definida por una ecuación de la forma $y = kx$, donde $k \neq 0$, representa una variación directa. |

**Example** $y = 18x$ is a direct variation.

| | |
|---|---|
| **Discrete graph (p. 70)** A graph composed of isolated points. | **Gráfica discreta (p. 70)** Una gráfica discreta es compuesta de puntos aislados. |

**Example**




| | |
|---|---|
| **Discriminant (p. 372)** The discriminant of a quadratic equation of the form $ax^2 + bx + c = 0$ is $b^2 - 4ac$. The value of the discriminant determines the number of solutions of the equation. | **Discriminante (p. 372)** El discriminante de una ecuación cuadrática $ax^2 + bx + c = 0$ es $b^2 - 4ac$. El valor del discriminante determina el número de soluciones de la ecuación. |

**Example** The discriminant of $2x^2 + 9x - 2 = 0$ is 97.

## English | Spanish

**Domain (of a relation or function) (p. 83)** The possible values for the input of a relation or function.

**Dominio (de una relación o función) (p. 83)** Posibles valores de entrada de una relación o función.

**Example** In the function $f(x) = x + 22$, the domain is all real numbers.

## E

**Elimination method (p. 175)** A method for solving a system of linear equations. You add or subtract the equations to eliminate a variable.

**Eliminación (p. 175)** Método para resolver un sistema de ecuaciones lineales. Se suman o se restan las ecuaciones para eliminar una variable.

**Example**

$$\begin{aligned} 3x + y &= 19 \\ \underline{2x - y} &\underline{{}= \ \ 1} \\ 5x + 0 &= 20 \end{aligned}$$

Add the equations to get $x = 4$.

$2(4) - y = 1 \longrightarrow$ Substitute 4 for $x$ in
$8 - y = 1$ the second equation.
$y = 7 \longrightarrow$ Solve for $y$.

**Excluded value (p. 305)** A value of $x$ for which a rational expression $f(x)$ is undefined.

**Valor excluido (p. 305)** Valor de $x$ para el cual una expresión racional es indefinida.

**Explicit formula (p. 246)** An explicit formula expresses the $n$th term of a sequence in terms of $n$.

**Fórmula explícita (p. 246)** Una fórmula explícita expresa el $n$-ésimo término de una progresión en función de $n$.

**Example** Let $a_n = 2n + 5$ for positive integers $n$. If $n = 7$, then $a_7 = 2(7) + 5 = 19$.

**Exponential decay (p. 395)** A situation modeled with a function of the form $y = ab^x$, where $a > 0$ and $0 < b < 1$.

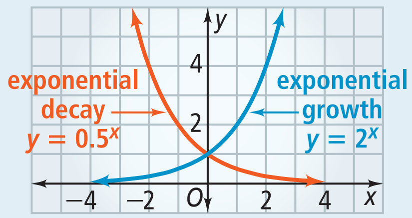
**Decremento exponencial (p. 395)** Para $a > 0$ y $0 < b < 1$, la función $y = ab^x$ representa el decremento exponencial.

**Example** $y = 5(0.1)^x$

**Exponential function (p. 386)** A function that repeatedly multiplies an initial amount by the same positive number. You can model all exponential functions using $y = ab^x$, where $a$ is a nonzero constant, $b > 0$, and $b \neq 1$.

**Función exponencial (p. 386)** Función que multiplica repetidas veces una cantidad inicial por el mismo número positivo. Todas las funciones exponenciales se pueden representar mediante $y = ab^x$, donde $a$ es una constante con valor distinto de cero, $b > 0$ y $b \neq 1$.

**Example**




**Exponential growth (p. 395)** A situation modeled with a function of the form $y = ab^x$, where $a > 0$ and $b > 1$.

**Incremento exponencial (p. 395)** Para $a > 0$ y $b > 1$, la función $y = ab^x$ representa el incremento exponencial.

**Example** $y = 100(2)^x$

## English | Spanish

**Extrapolation (p. 144)** The process of predicting a value outside the range of known values.

**Extrapolación (p. 144)** Proceso que se usa para predecir un valor por fuera del ámbito de los valores dados.

## F

**Factor by grouping (p. 301)** A method of factoring that uses the Distributive Property to remove a common binomial factor of two pairs of terms.

**Factor común por agrupación de términos (p. 301)** Método de factorización que aplica la propiedad distributiva para sacar un factor común de dos pares de términos en un binomio.

**Example** The expression $7x(x - 1) + 4(x - 1)$ can be factored as $(7x + 4)(x - 1)$.

**Falling object model (p. 326)** The function $h = -16t^2 + c$ models the height of a falling object, where $h$ is the object's height in feet, $t$ is the time in seconds since the object began to fall, and $c$ is the object's initial height.

**Modelo de objetos que caen (p. 326)** La función $h = -16t^2 + c$ representa la altura de un objeto que cae, donde $h$ es la altura del objeto en pies, $t$ es el tiempo de caída en segundos y $c$ es la altura inicial del objeto.

**Formula (p. 17)** An equation that states a relationship among quantities.

**Fórmula (p. 17)** Ecuación que establece una relación entre cantidades.

**Example** The formula for the volume $V$ of a cylinder is $V = \pi r^2 h$, where $r$ is the radius of the cylinder and $h$ is its height.

**Function (p. 59)** A relation that assigns exactly one value in the range to each value of the domain.

**Función (p. 59)** La relación que asigna exactamente un valor del rango a cada valor del dominio.

**Example** Earned income is a function of the number of hours worked. If you earn \$4.50/h, then your income is expressed by the function $f(h) = 4.5h$.

**Function notation (p. 88)** To write a rule in function notation, you use the symbol $f(x)$ in place of $y$.

**Notación de una función (p. 88)** Para expresar una regla en notación de función se usa el símbolo $f(x)$ en lugar de $y$.

**Example** $f(x) = 3x - 8$ is in function notation.

## G

**Geometric sequence (p. 246)** A number sequence formed by multiplying a term in a sequence by a fixed number to find the next term.

**Progresión geométrica (p. 246)** Tipo de sucesión numérica formada al multiplicar un término de la secuencia por un número constante, para hallar el siguiente término.

**Example** $9, 3, 1, \frac{1}{3}, \ldots$ is an example of a geometric sequence.

# English | Spanish

**Growth factor (p. 395)** 1 plus the percent rate of change for an exponential growth situation.

**Factor incremental (p. 395)** 1 más la tasa porcentual de cambio en una situación de incremento exponencial.

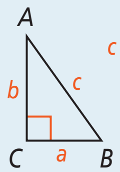
**Example** The growth factor of $y = 7(1.3)^x$ is 1.3.

## H

**Hypotenuse (p. 233)** The side opposite the right angle in a right triangle. It is the longest side in the triangle.

**Hipotenusa (p. 233)** En un triángulo rectángulo, el lado opuesto al ángulo recto. Es el lado más largo del triángulo.

**Example**




$c$ **is the hypotenuse.**

## I

**Identity (p. 11)** An equation that is true for every value.

**Identidad (p. 11)** Una ecuación que es verdadera para todos los valores.

**Example** $5 - 14x = 5\left(1 - \frac{14}{5}x\right)$ is an identity because it is true for any value of $x$.

**Inconsistent system (p. 160)** A system of equations that has no solution.

**Sistema incompatible (p. 160)** Un sistema incompatible es un sistema de ecuaciones para el cual no hay solución.

**Example** $\begin{cases} y = 2x + 3 \\ -2x + y = 1 \end{cases}$ is a system of parallel lines, so it has no solution. It is an inconsistent system.

**Independent system (p. 160)** A system of linear equations that has a unique solution.

**Sistema independiente (p. 160)** Un sistema de ecuaciones lineales que tenga una sola solución es un sistema independiente.

**Example** $\begin{cases} x + 2y = -7 \\ 2x - 3y = 0 \end{cases}$ has the unique solution $(-3, -2)$. It is an independent system.

**Independent variable (p. 59)** A variable that provides the input values of a function.

**Variable independiente (p. 59)** Variable de la que dependen los valores de entrada de una función.

**Example** In the equation $y = 3x$, $x$ is the independent variable.

## English | Spanish

**Index (p. 225)** With a radical sign, the index indicates the degree of the root.

**Índice (p. 225)** Con un signo de radical, el índice indica el grado de la raíz.

**Example** index 2 $\sqrt{16}$ index 3 $\sqrt[3]{16}$ index 4 $\sqrt[4]{16}$

---

**Input (p. 59)** A value of the independent variable.

**Entrada (p. 59)** Valor de una variable independiente.

**Example** The input is any value of $x$ you substitute into a function.

---

**Interpolation (p. 144)** The process of estimating a value between two known quantities.

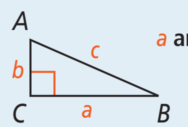
**Interpolación (p. 144)** Proceso que se usa para estimar el valor entre dos cantidades dadas.

## L

**Leg (p. 233)** Each of the sides that form the right angle of a right triangle.

**Cateto (p. 233)** Cada uno de los dos lados que forman el ángulo recto en un triángulo rectángulo.

**Example**



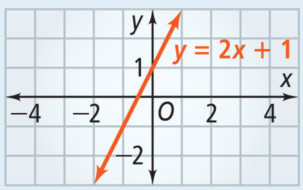

$a$ and $b$ are legs.

---

**Linear equation (p. 115)** An equation whose graph forms a straight line.

**Ecuación lineal (p. 115)** Ecuación cuya gráfica es una línea recta.

**Example**

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**Linear factor (p. 360)** A linear factor of an expression is a factor in the form $ax + b$, $a \neq 0$.

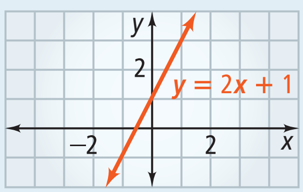
**Factor lineal (p. 360)** Un factor lineal de una expresión es un factor que tiene la forma $ax + b$, $a \neq 0$.

---

**Linear function (p. 59)** A function whose graph is a line is a linear function. You can represent a linear function with a linear equation.

**Función lineal (p. 59)** Una función cuya gráfica es una recta es una función lineal. La función lineal se representa con una ecuación lineal.

**Example**



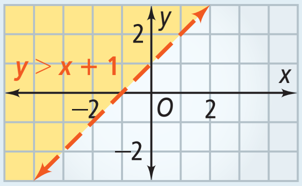
## English

## Spanish

**Linear inequality (p. 187)** An inequality in two variables whose graph is a region of the coordinate plane that is bounded by a line. Each point in the region is a solution of the inequality.

**Desigualdad lineal (p. 187)** Una desigualdad lineal es una desigualdad de dos variables cuya gráfica es una región del plano de coordenadas delimitado por una recta. Cada punto de la región es una solución de la desigualdad.

**Example**




**Linear parent function (p. 115)** The simplest form of a linear function.

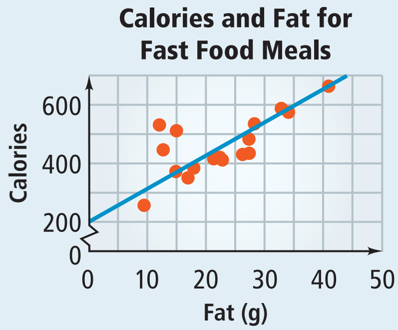
**Función lineal elemental (p. 115)** La forma más simple de una función lineal.

**Example** $y = x$

**Line of best fit (p. 144)** The most accurate trend line on a scatter plot showing the relationship between two sets of data.

**Recta de mayor aproximación (p. 144)** La línea de tendencia en un diagrama de puntos que más se acerca a los puntos que representan la relación entre dos conjuntos de datos.

**Example**




**Literal equation (p. 17)** An equation involving two or more variables.

**Ecuación literal (p. 17)** Ecuación que incluye dos o más variables.

**Example** $4x + 2y = 18$ is a literal equation.

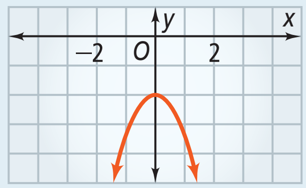
## English | Spanish

### M

**Maximum (p. 326)** The *y*-coordinate of the vertex of a parabola that opens downward.

**Máximo (p. 326)** La coordenada *y* del vértice en una parábola que se abre hacia abajo.

**Example**

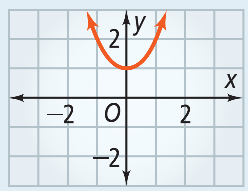



Since the parabola opens downward, the *y*-coordinate of the vertex is the function's maximum value.

**Minimum (p. 326)** The *y*-coordinate of the vertex of a parabola that opens upward.

**Mínimo (p. 326)** La coordenada *y* del vértice en una parábola que se abre hacia arriba.

**Example**




Since the parabola opens upward, the *y*-coordinate of the vertex is the function's minimum value.

**Monomial (p. 268)** A real number, a variable, or a product of a real number and one or more variables with whole-number exponents.

**Monomio (p. 268)** Número real, variable o el producto de un número real y una o más variables con números enteros como exponentes.

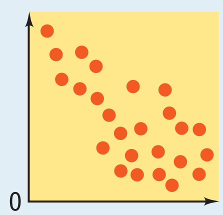
**Example** 9, $n$, and $-5xy^2$ are examples of monomials.

### N

**Negative correlation (p. 144)** The relationship between two sets of data, in which one set of data decreases as the other set of data increases.

**Correlación negativa (p. 144)** Relación entre dos conjuntos de datos en la que uno de los conjuntos disminuye a medida que el otro aumenta.

**Example**

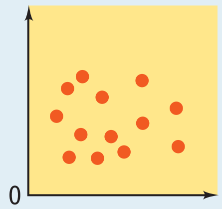


| English | Spanish |
| --- | --- |

**No correlation (p. 144)** There does not appear to be a relationship between two sets of data.

**Sin correlación (p. 144)** No hay relación entre dos conjuntos de datos.

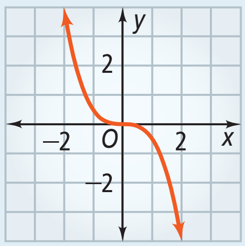
**Example**




**Nonlinear function (p. 64)** A function whose graph is not a line or part of a line.

**Función no lineal (p. 64)** Función cuya gráfica no es una línea o parte de una línea.

**Example**




***n*th root (p. 225)** For any real numbers $a$ and $b$, and any positive integer $n$, if $a^n = b$, then $a$ is an $n$th root of $b$.

**raíz *n*-ésima (p. 225)** Para todos los números reales $a$ y $b$, y todo número entero positivo $n$, si $a^n = b$, entonces $a$ es la $n$-ésima raíz de $b$.

**Example** $\sqrt[5]{32} = 2$ because $2^5 = 32$.
$\sqrt[4]{81} = 3$ because $3^4 = 81$.

## O

**Opposite reciprocals (p. 132)** A number of the form $-\frac{b}{a}$, where $\frac{a}{b}$ is a nonzero rational number. The product of a number and its opposite reciprocal is $-1$.

**Recíproco inverso (p. 132)** Número en la forma $-\frac{b}{a}$, donde $\frac{a}{b}$ es un número racional diferente de cero. El producto de un número y su recíproco inverso es $-1$.

**Example** $\frac{2}{5}$ and $-\frac{5}{2}$ are opposite reciprocals because $\left(\frac{2}{5}\right)\left(-\frac{5}{2}\right) = -1$.

**Output (p. 59)** A value of the dependent variable.

**Salida (p. 59)** Valor de una variable dependiente.

**Example** The output of the function $f(x) = x^2$ when $x = 3$ is 9.

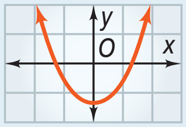
# English / Spanish

## P

**Parabola (p. 326)** The graph of a quadratic function.

**Parábola (p. 326)** La gráfica de una función cuadrática.

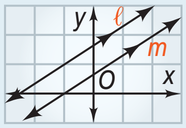
**Example**




**Parallel lines (p. 132)** Two lines in the same plane that never intersect. Parallel lines have the same slope.

**Rectas paralelas (p. 132)** Dos rectas situadas en el mismo plano que nunca se cortan. Las rectas paralelas tienen la misma pendiente.

**Example**




**Parent function (p. 115)** A family of functions is a group of functions with common characteristics. A parent function is the simplest function with these characteristics.

**Función elemental (p. 115)** Una familia de funciones es un grupo de funciones con características en común. La función elemental es la función más simple que reúne esas características.

**Example** $y = x$ is the parent function for the family of linear equations of the form $y = mx + b$.

**Perfect square trinomial (p. 296)** Any trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$.

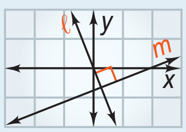
**Trinomio cuadrado perfecto (p. 296)** Todo trinomio de la forma $a^2 + 2ab + b^2$ ó $a^2 - 2ab + b^2$.

**Example** $(x + 3)^2 = x^2 + 6x + 9$

**Perpendicular lines (p. 132)** Lines that intersect to form right angles. Two lines are perpendicular if the product of their slopes is $-1$.

**Rectas perpendiculares (p. 132)** Rectas que forman ángulos rectos en su intersección. Dos rectas son perpendiculares si el producto de sus pendientes es $-1$.

**Example**




**Point-slope form (p. 120)** A linear equation of a nonvertical line written as $y - y_1 = m(x - x_1)$. The line passes through the point $(x_1, y_1)$ with slope $m$.

**Forma punto-pendiente (p. 120)** La ecuación lineal de una recta no vertical que pasa por el punto $(x_1, y_1)$ con pendiente $m$ está dada por $y - y_1 = m(x - x_1)$.

**Example** An equation with a slope of $-\frac{1}{2}$ passing through $(2, -1)$ would be written $y + 1 = -\frac{1}{2}(x - 2)$ in point-slope form.

Visual Glossary

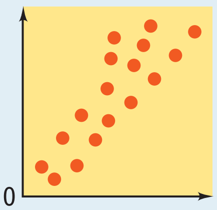
| English | Spanish |
|---|---|
| **Polynomial (p. 268)** A monomial or the sum or difference of two or more monomials. A quotient with a variable in the denominator is not a polynomial. | **Polinomio (p. 268)** Un monomio o la suma o diferencia de dos o más monomios. Un cociente con una variable en el denominador no es un polinomio. |

**Example** $2x^2$, $3x + 7$, 28, and $-7x^3 - 2x^2 + 9$ are all polynomials.

**Positive correlation (p. 144)** The relationship between two sets of data in which both sets of data increase together.

**Correlación positiva (p. 144)** La relación entre dos conjuntos de datos en la que ambos conjuntos incrementan a la vez.

**Example**




**Principal square root (p. 225)** A number of the form $\sqrt{b}$. The expression $\sqrt{b}$ is called the principal (or positive) square root of $b$.

**Raíz cuadrada principal (p. 225)** La expresión $\sqrt{b}$ se llama raíz cuadrada principal (o positiva) de $b$.

**Example** 5 is the principal square root of $\sqrt{25}$.

**Proportion (p. 22)** An equation that states that two ratios are equal.

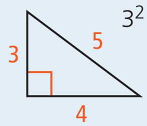
**Proporción (p. 22)** Es una ecuación que establece que dos razones son iguales.

**Example** $\frac{7.5}{9} = \frac{5}{6}$

**Pythagorean Theorem (p. 233)** In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse: $a^2 + b^2 = c^2$.

**Teorema de Pitágoras (p. 233)** En un triángulo rectángulo, la suma de los cuadrados de los catetos es igual al cuadrado de la hipotenusa: $a^2 + b^2 = c^2$.

**Example**




$3^2 + 4^2 = 5^2$

## Q

**Quadratic equation (p. 350)** A quadratic equation is one that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

**Ecuación cuadrática (p. 350)** Ecuación que puede expresarse de la forma normal como $ax^2 + bx + c = 0$, en la que $a \neq 0$.

**Example** $4x^2 + 9x - 5 = 0$

# English

# Spanish

**Quadratic formula (p. 372)** If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Fórmula cuadrática (p. 372)** Si $ax^2 + bx + c = 0$ y $a \neq 0$, entonces $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Example** $2x^2 + 10x + 12 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-10 \pm \sqrt{10^2 - 4(2)(12)}}{2(2)}$$
$$x = \frac{-10 \pm \sqrt{4}}{4}$$
$$x = \frac{-10 + 2}{4} \text{ or } \frac{-10 - 2}{4}$$
$$x = -2 \text{ or } -3$$

**Quadratic function (p. 326)** A function of the form $y = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is a parabola, a U-shaped curve that opens up or down.

**Función cuadrática (p. 326)** La función $y = ax^2 + bx + c$, en la que $a \neq 0$. La gráfica de una función cuadrática es una parábola, o curva en forma de U que se abre hacia arriba o hacia abajo.

**Example** $y = 5x^2 - 2x + 1$ is a quadratic function.

**Quadratic parent function (p. 326)** The simplest quadratic function $f(x) = x^2$ or $y = x^2$.

**Función cuadrática madre (p. 326)** La función cuadrática más simple $f(x) = x^2$ ó $y = x^2$.

**Example** $y = x^2$ is the parent function for the family of quadratic equations of the form $y = ax^2 + bx + c$.

## R

**Radical (p. 225)** An expression made up of a radical symbol and a radicand.

**Radical (p. 225)** Expresión compuesta por un símbolo radical y un radicando.

**Example** $\sqrt{a}$

**Radical expression (p. 225)** Expression that contains a radical.

**Expresión radical (p. 225)** Expresiones que contienen radicales.

**Example** $\sqrt{3}$, $\sqrt{5x}$, and $\sqrt{x - 10}$ are examples of radical expressions.

**Radicand (p. 225)** The expression under the radical sign is the radicand.

**Radicando (p. 225)** La expresión que aparece debajo del signo radical es el radicando.

**Example** The radicand of the radical expression $\sqrt{x + 2}$ is $x + 2$.

## English | Spanish

**Range (of a relation or function) (p. 83)** The possible values of the output, or dependent variable, of a relation or function.

**Rango (de una relación o función) (p. 83)** El conjunto de todos los valores posibles de la salida, o variable dependiente, de una relación o función.

**Example** In the function $y = |x|$, the range is the set of all nonnegative numbers.

**Rate (p. 22)** A ratio of $a$ to $b$ where $a$ and $b$ represent quantities measured in different units.

**Tasa (p. 22)** La relación que existe entre $a$ y $b$ cuando $a$ y $b$ son cantidades medidas con distintas unidades.

**Example** Traveling 125 miles in 2 hours results in the rate $\frac{125 \text{ miles}}{2 \text{ hours}}$ or 62.5 mi/h.

**Rate of change (p. 100)** The relationship between two quantities that are changing. The rate of change is also called slope.

$$\text{rate of change} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

**Tasa de cambio (p. 100)** La relación entre dos cantidades que cambian. La tasa de cambio se llama también pendiente.

$$\text{tasa de cambio} = \frac{\text{cambio en la variable dependiente}}{\text{cambio en la variable independiente}}$$

**Example** Video rental for 1 day is $1.99. Video rental for 2 days is $2.99.

$$\begin{aligned}\text{rate of change} &= \frac{2.99 - 1.99}{2 - 1}\\ &= \frac{1.00}{1}\\ &= 1\end{aligned}$$

**Ratio (p. 22)** A ratio is the comparison of two quantities by division.

**Razón (p. 22)** Una razón es la comparación de dos cantidades por medio de una división.

**Example** $\frac{5}{7}$ and 7 : 3 are ratios.

**Rational expression (p. 305)** A ratio of two polynomials. The value of the variable cannot make the denominator equal to 0.

**Expresión racional (p. 305)** Una razón de dos polinomios. El valor de la variable no puede hacer el denominador igual a 0.

**Example** $\frac{3}{x^3 + x}$, where $x \neq 0$

**Rationalize the denominator (p. 229)** To rationalize the denominator of an expression, rewrite it so there are no radicals in any denominator and no denominators in any radical.

**Racionalizar el denominador (p. 229)** Para racionalizar el denominador de una expresión, ésta se escribe de modo que no haya radicales en ningún denominador y no haya denominadores en ningún radical.

**Example** $\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}$

**Recursive formula (p. 251)** A recursive formula defines the terms in a sequence by relating each term to the ones before it.

**Fórmula recursiva (p. 251)** Una fórmula recursiva define los términos de una secuencia al relacionar cada término con los términos que lo anteceden.

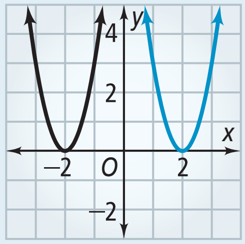
**Example** Let $A(n) = 2.5A(n - 1) + 3A(n - 2)$.
If $A(5) = 3$ and $A(4) = 7.5$, then
$A(6) = 2.5(3) + 3(7.5) = 30$.

## English | Spanish

**Reflection (p. 137)** A reflection flips the graph of a function across a line, such as the *x*- or *y*-axis. Each point on the graph of the reflected function is the same distance from the line of reflection as is the corresponding point on the graph of the original function.

**Reflexión (p. 137)** Una reflexión voltea la gráfica de una función sobre una línea, como el eje de las *x* o el eje de las *y*. Cada punto de la gráfica de la función reflejada está a la misma distancia del eje de reflexión que el punto correspondiente en la gráfica de la función original.

**Example**




**Relation (p. 83)** Any set of ordered pairs.

**Relación (p. 83)** Cualquier conjunto de pares ordenados.

**Example** {(0, 0), (2, 3), (2, −7)} is a relation.

**Root of an equation (p. 350)** A solution of an equation.

**Ráiz de una ecuación (p. 350)** Solucion de una ecuación.

## S

**Scale (p. 27)** The ratio of any length in a scale drawing to the corresponding actual length. The lengths may be in different units.

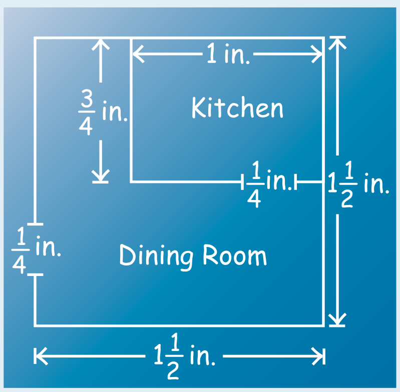
**Escala (p. 27)** Razón de cualquier longitud de un dibujo a escala a la longitud real correspondiente. Las longitudes pueden tener diferentes unidades.

**Example** For a drawing in which a 2-in. length represents an actual length of 18 ft, the scale is 1 in. : 9 ft.

**Scale drawing (p. 27)** An enlarged or reduced drawing similar to an actual object or place.

**Dibujo a escala (p. 27)** Dibujo que muestra de mayor o menor tamaño un objeto o lugar dado.

**Example**



## English

## Spanish

**Scale model (p. 27)** A three-dimensional model that is similar to a three-dimensional object.

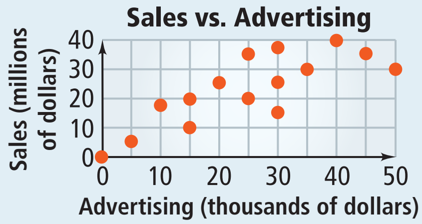
**Modelo de escala (p. 27)** Modelo tridimensional que es similar a un objeto tridimensional.

**Example** A ship in a bottle is a scale model of a real ship.

**Scatter plot (p. 144)** A graph that relates two different sets of data by displaying them as ordered pairs.

**Diagrama de puntos (p. 144)** Gráfica que muestra la relación entre dos conjuntos. Los datos de ambos conjuntos se presentan como pares ordenados.

**Example**




The scatter plot displays the amount spent on advertising (in thousands of dollars) versus product sales (in millions of dollars).

**Sequence (p. 246)** An ordered list of numbers that often forms a pattern.

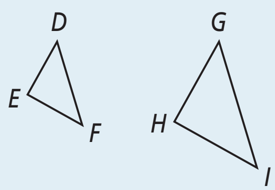
**Progresión (p. 246)** Lista ordenada de números que muchas veces forma un patrón.

**Example** −4, 5, 14, 23 is a sequence.

**Similar figures (p. 27)** Similar figures are two figures that have the same shape, but not necessarily the same size.

**Figuras semejantes (p. 27)** Dos figuras semejantes son dos figuras que tienen la misma forma pero no son necesariamente del mismo tamaño.

**Example**




$\triangle DEF$ and $\triangle GHI$ are similar.

| English | Spanish |
|---|---|

## English

## Spanish

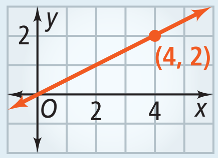
**Slope (p. 100)** The ratio of the vertical change to the horizontal change.

$\text{slope} = \dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_2 - x_1 \neq 0$

**Pendiente (p. 100)** La razón del cambio vertical al cambio horizontal.

$\text{pendiente} = \dfrac{\text{cambio vertical}}{\text{cambio horizontal}} = \dfrac{y_2 - y_1}{x_2 - x_1}$, donde $x_2 - x_1 \neq 0$

**Example**




The slope of the line above is $\frac{2}{4} = \frac{1}{2}$.

**Slope-intercept form (p. 115)** The slope-intercept form of a linear equation is $y = mx + b$, where $m$ is the slope of the line and $b$ is the $y$-intercept.

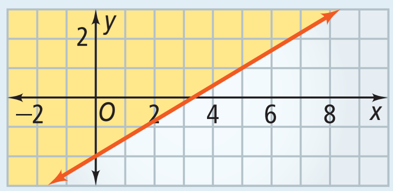
**Forma pendiente-intercepto (p. 115)** La forma pendiente-intercepto es la ecuación lineal $y = mx + b$, en la que $m$ es la pendiente de la recta y $b$ es el punto de intersección de esa recta con el eje $y$.

**Example** $y = 8x - 2$

**Solution of an inequality (two variables) (p. 187)** Any ordered pair that makes the inequality true.

**Solución de una desigualdad (dos variables) (p. 187)** Cualquier par ordenado que haga verdadera la desigualdad.

**Example** Each ordered pair in the yellow area and on the solid red line is a solution of $3x - 5y \leq 10$.




**Solution of a system of linear equations (p. 160)** Any ordered pair in a system that makes all the equations of that system true.

**Solución de un sistema de ecuaciones lineales (p. 160)** Todo par ordenado de un sistema que hace verdaderas todas las ecuaciones de ese sistema.

**Example** (2, 1) is a solution of the system

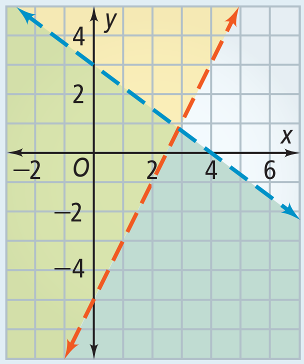
$y = 2x - 3$
$y = x - 1$

because the ordered pair makes both equations true.

## English

**Solution of a system of linear inequalities (p. 194)** Any ordered pair that makes all of the inequalities in the system true.

**Example**




The shaded green area shows the solution of the system $\begin{matrix} y > 2x - 5 \\ 3x + 4y < 12 \end{matrix}$.

**Standard form of a linear equation (p. 125)** The standard form of a linear equation is $Ax + By = C$, where $A$, $B$, and $C$ are real numbers and $A$ and $B$ are not both zero.

**Example** $6x - y = 12$

**Standard form of a polynomial (p. 268)** The form of a polynomial that places the terms in descending order by degree.

**Example** $15x^3 + x^2 + 3x + 9$

**Standard form of a quadratic equation (p. 350)** The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$.

**Example** $-x^2 + 2x - 9 = 0$

**Standard form of a quadratic function (p. 326)** The standard form of a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$.

**Example** $f(x) = 2x^2 - 5x + 2$

**Stretch (p. 340)** A stretch is a transformation that increases the distance between corresponding points of a graph.

## Spanish

**Solución de un sistema de desigualdades lineales (p. 194)** Todo par ordenado que hace verdaderas todas las desigualdades del sistema.

**Forma normal de una ecuación lineal (p. 125)** La forma normal de una ecuación lineal es $Ax + By = C$, donde $A$, $B$ y $C$ son números reales, y donde $A$ y $B$ no son iguales a cero.

**Forma normal de un polinomio (p. 268)** Cuando el grado de los términos de un polinomio disminuye de izquierda a derecha, está en forma normal, o en orden descendente.

**Forma normal de una ecuación cuadrática (p. 350)** Cuando una ecuación cuadrática se expresa de forma $ax^2 + bx + c = 0$.

**Forma normal de una función cuadrática (p. 326)** La forma normal de una función cuadrática es $f(x) = ax^2 + bx + c$, donde $a \neq 0$.

**Estiramiento (p. 340)** Un estiramiento es una transformación que aumenta la distancia entre puntos que se corresponden en una gráfica.

| English | Spanish |
|---|---|
| **Substitution method (p. 169)** A method of solving a system of equations by replacing one variable with an equivalent expression containing the other variable. | **Método de sustitución (p. 169)** Método para resolver un sistema de ecuaciones en el que se reemplaza una variable por una expresión equivalente que contenga la otra variable. |

**Example** If $y = 2x + 5$ and $x + 3y = 7$, then $x + 3(2x + 5) = 7$.

| | |
|---|---|
| **System of linear equations (p. 160)** Two or more linear equations using the same variables. | **Sistema de ecuaciones lineales (p. 160)** Dos o más ecuaciones lineales que usen las mismas variables. |

**Example** $y = 5x + 7$
$y = \frac{1}{2}x - 3$

| | |
|---|---|
| **System of linear inequalities (p. 194)** Two or more linear inequalities using the same variables. | **Sistema de desigualdades lineales (p. 194)** Dos o más desigualdades lineales que usen las mismas variables. |

**Example** $y \leq x + 11$
$y < 5x$

## T

| | |
|---|---|
| **Term of a sequence (p. 246)** A term of a sequence is any number in a sequence. | **Término de una progresión (p. 246)** Un término de una secuencia es cualquier número de una secuencia. |

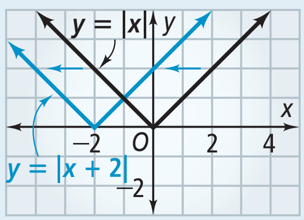
**Example** $-4$ is the first term of the sequence $-4, 5, 14, 23$.

| | |
|---|---|
| **Transformation (p. 137)** A transformation of a function $y = af(x - h) + k$ is a change made to at least one of the values $a$, $h$, and $k$. The four types of transformations are dilations, reflections, rotations, and translations. | **Transformación (p. 137)** Una transformación de una función $y = af(x - h) + k$ es un cambio que se le hace a por lo menos uno de los valores $a$, $h$ y $k$. Hay cuatro tipos de transformaciones: dilataciones, reflexiones, rotaciones y traslaciones. |

**Example** $g(x) = 2(x - 3)$ is a transformation of $f(x) = x$.

| | |
|---|---|
| **Translation (p. 137)** A transformation that shifts a graph horizontally, vertically, or both. | **Translación (p. 137)** Proceso de mover una gráfica horizontalmente, verticalmente o en ambos sentidos. |

**Example**




$y = |x + 2|$ is a translation of $y = |x|$.



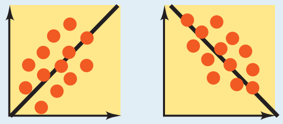
Visual Glossary

## English / Spanish

**Trend line (p. 144)** A line on a scatter plot drawn near the points. It shows a correlation.

**Línea de tendencia (p. 144)** Línea de un diagrama de puntos que se traza cerca de los puntos para mostrar una correlación.

**Example**




**Trinomial (p. 268)** A polynomial of three terms.

**Trinomio (p. 268)** Polinomio compuesto de tres términos.

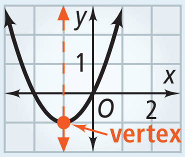
**Example** $3x^2 + 2x - 5$

## V

**Vertex (p. 326)** The highest or lowest point on a parabola. The axis of symmetry intersects the parabola at the vertex.

**Vértice (p. 326)** El punto más alto o más bajo de una parábola. El punto de intersección del eje de simetría y la parábola.

**Example**




**Vertex form of a quadratic function (p. 346)** The vertex form of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$ and $(h, k)$ is the coordinate of the vertex of the function.

**Forma del vértice de una función cuadrática (p. 346)** La forma vértice de una función cuadrática es $f(x) = a(x - h)^2 + k$, donde $a \neq 0$ y $(h, k)$ es la coordenada del vértice de la función.

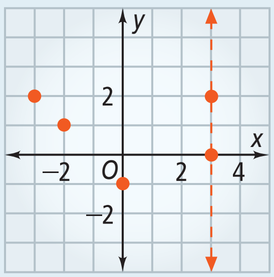
**Example** $f(x) = x^2 + 2x - 1 = (x + 1)^2 - 2$

The vertex is $(-1, -2)$.

**Vertical-line test (p. 83)** The vertical-line test is a method used to determine if a relation is a function or not. If a vertical line passes through a graph more than once, the graph is not the graph of a function.

**Prueba de la recta vertical (p. 83)** La prueba de recta vertical es un método que se usa para determinar si una relación es una función o no. Si una recta vertical pasa por el medio de una gráfica más de una vez, la gráfica no es una gráfica de una función.

**Example**




A line would pass through (3, 0) and (3, 2), so the relation is not a function.

Visual Glossary

## English | Spanish

**Vertical motion model (p. 333)** The function $h = -16t^2 + vt + c$ models the height of a object, where $h$ is the object's height in feet, $t$ is the time in seconds since the object began to fall, $v$ is the object's initial velocity, and $c$ is the object's initial height.

**Modelo de movimiento vertical (p. 333)** La función $h = -16t^2 + vt + c$ representa la altura de un objeto donde $h$ es la altura del objeto en pies, $t$ es el tiempo de caída en segundos, $v$ es la velocidad inicial del objeto y $c$ es la altura inicial del objeto.

## X

***x*-intercept (p. 125)** The $x$-coordinate of a point where a graph crosses the $x$-axis.

**Intercepto en *x* (p. 125)** Coordenada $x$ por donde la gráfica cruza el eje de las $x$.

**Example** The $x$-intercept of $3x + 4y = 12$ is 4.

## Y

***y*-intercept (p. 115)** The $y$-coordinate of a point where a graph crosses the $y$-axis.

**Intercepto en *y* (p. 115)** Coordenada $y$ por donde la gráfica cruza el eje de las $y$.

**Example** The $y$-intercept of $y = 5x + 2$ is 2.

## Z

**Zero-Product Property (p. 356)** For all real numbers $a$ and $b$, if $ab = 0$, then $a = 0$ or $b = 0$.

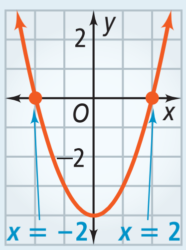
**Propiedad del producto cero (p. 356)** Para todos los números reales $a$ y $b$, si $ab = 0$, entonces $a = 0$ ó $b = 0$.

**Example**
$x(x + 3) = 0$
$x = 0$ or $x + 3 = 0$
$x = 0$ or $x = -3$

**Zero of a function (p. 350)** An $x$-intercept of the graph of a function.

**Cero de una función (p. 350)** Intercepto $x$ de la gráfica de una función.

**Example** The zeros of $y = x^2 - 4$ are $\pm 2$.



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# Acknowledgments

## Cover Design

Tom White / 9 Surf Studios
Brian Reardon and Jamee Farinella

## Photographs

Photo locators denoted as follows: Top (T), Center (C), Bottom (B), Left (L), Right (R), Background (Bkgd)

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