

Name _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the exact value of the expression.

1) $\sin^{-1} \frac{\sqrt{3}}{2}$ 1) _____

2) $\cos^{-1} \frac{\sqrt{3}}{2}$ 2) _____

3) $\tan^{-1}(-1)$ 3) _____

Solve the problem.

4) The formula 4) _____

$$D = 24 \left[1 - \frac{\cos^{-1}(\tan i \tan \theta)}{\pi} \right]$$

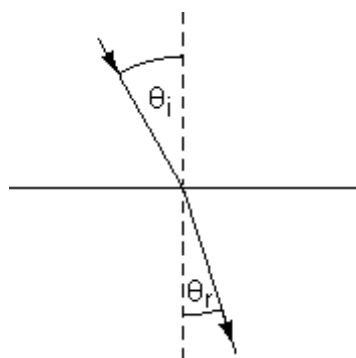
can be used to approximate the number of hours of daylight when the declination of the sun is i° at a location θ° north latitude for any date between the vernal equinox and autumnal equinox. To use this formula, $\cos^{-1}(\tan i \tan \theta)$ must be expressed in radians. Approximate the number of hours of daylight in Flagstaff, Arizona, ($35^\circ 13'$ north latitude) for summer solstice ($i = 23.5^\circ$).

- 5) When light travels from one medium to another—from air to water, for instance—it changes direction. (This is why a pencil, partially submerged in water, looks as though it is bent.) The angle of incidence θ_i is the angle in the first medium; the angle of refraction θ_r is the angle in the second medium. (See illustration.) Each medium has an index of refraction— n_i and n_r , respectively—which can be found in tables. Snell's law relates these quantities in the formula

$$n_i \sin \theta_i = n_r \sin \theta_r$$

Solving for θ_r , we obtain

$$\theta_r = \sin^{-1} \left(\frac{n_i}{n_r} \sin \theta_i \right)$$

Find θ_r for crown glass ($n_i = 1.52$), water ($n_r = 1.33$), and $\theta_i = 38^\circ$.

Find the exact value of the expression. Do not use a calculator.

6) $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$ 6) _____

7) $\cos^{-1}\left[\cos\left(-\frac{\pi}{4}\right)\right]$ 7) _____

8) $\sin^{-1}\left(\sin \frac{7\pi}{6}\right)$ 8) _____

Find the exact value, if any, of the composite function. If there is no value, say it is "not defined". Do not use a calculator.

9) $\sin(\sin^{-1} 1.8)$ 9) _____

10) $\tan(\tan^{-1}(-9.1))$ 10) _____

11) $\cos[\cos^{-1}(-1.3)]$ 11) _____

Find the inverse function f^{-1} of the function f .

12) $f(x) = 9 \tan(8x)$ 12) _____

13) $f(x) = -\sin(x + 9) - 3$ 13) _____

14) $f(x) = -9 \cos(6x)$ 14) _____

Find the domain of the function f and of its inverse function f^{-1} .

15) $f(x) = 2 \sin(4x)$ 15) _____

16) $f(x) = \cos(x - 3) + 7$ 16) _____

17) $f(x) = \tan(x - 6) + 5$ 17) _____

Find the exact solution of the equation.

18) $4 \cos^{-1}(5x) = \pi$ 18) _____

19) $3 \tan^{-1}(2x) = \pi$ 19) _____

20) $5 \sin^{-1} x - 4\pi = 3 \sin^{-1} x - 5\pi$ 20) _____

Find the exact value of the expression.

21) $\sin\left[\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$ 21) _____

22) $\cot[\sin^{-1}(-1)]$ 22) _____

$$23) \cos\left(\sin^{-1} \frac{1}{2}\right) \quad 23) \underline{\hspace{2cm}}$$

$$24) \sec[\tan^{-1}(-\sqrt{3})] \quad 24) \underline{\hspace{2cm}}$$

$$25) \sin^{-1}\left(\sin \frac{5\pi}{7}\right) \quad 25) \underline{\hspace{2cm}}$$

$$26) \cos^{-1}\left[\sin\left(\frac{5\pi}{4}\right)\right] \quad 26) \underline{\hspace{2cm}}$$

Use a calculator to find the value of the expression in radian measure rounded to two decimal places.

$$27) \csc^{-1}(-7) \quad 27) \underline{\hspace{2cm}}$$

$$28) \sec^{-1}\left(\frac{7}{3}\right) \quad 28) \underline{\hspace{2cm}}$$

$$29) \cot^{-1}\left(\frac{5}{11}\right) \quad 29) \underline{\hspace{2cm}}$$

Write the trigonometric expression as an algebraic expression in u .

$$30) \sin(\tan^{-1} u) \quad 30) \underline{\hspace{2cm}}$$

$$31) \cos(\cot^{-1} u) \quad 31) \underline{\hspace{2cm}}$$

Solve the equation on the interval $0 \leq \theta < 2\pi$.

$$32) \cos^2 \theta + 2 \cos \theta + 1 = 0 \quad 32) \underline{\hspace{2cm}}$$

$$33) 2 \sin^2 \theta - 3 \sin \theta - 2 = 0 \quad 33) \underline{\hspace{2cm}}$$

$$34) 3 \cot^2 \theta - 4 \csc \theta = 1 \quad 34) \underline{\hspace{2cm}}$$

$$35) 2 \sin^2 \theta = 3(\cos \theta + 1) \quad 35) \underline{\hspace{2cm}}$$

$$36) (\csc \theta - 2)(\cot \theta + 1) = 0 \quad 36) \underline{\hspace{2cm}}$$

Solve the problem.

$$37) \text{ The altitude of a projectile in feet (neglecting air resistance) is given by} \quad 37) \underline{\hspace{2cm}}$$

$$y = (\tan \theta)x - \frac{16}{v^2 \cos^2 \theta} x^2,$$

where x is the horizontal distance covered in feet and v is the initial velocity of the projectile at an angle θ from the horizontal. Find the firing angle (in degrees) of a projectile fired at an initial velocity of 100 feet per second so that it strikes the ground 312.5 feet from the firing point.

Simplify the trigonometric expression by following the indicated direction.

38) Factor and simplify: $\frac{2 \sin^2 \theta + 3 \sin \theta + 1}{\sin^2 \theta - 1}$

38) _____

Simplify the expression.

39) $(1 + \cot \theta)(1 - \cot \theta) - \csc^2 \theta$

39) _____

Establish the identity.

40) $(\sec u - \tan u)(\sec u + \tan u) = 1$

40) _____

41) $9 \csc^2 \theta - 6 \cot^2 \theta = 3 \csc^2 \theta + 6$

41) _____

42) $\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} = 2 \tan^2 x$

42) _____

43) $1 + \sec^2 x \sin^2 x = \sec^2 x$

43) _____

Answer Key

Testname: UNTITLED1

1) $\frac{\pi}{3}$

2) $\frac{\pi}{6}$

3) $-\frac{\pi}{4}$

4) 14.38 hr

5) $\theta_r = 44.72^\circ$

6) $\frac{2\pi}{5}$

7) $\frac{\pi}{4}$

8) $-\frac{\pi}{6}$

9) not defined

10) -9.1

11) not defined

12) $f^{-1}(x) = \frac{1}{8} \tan^{-1}\left(\frac{x}{9}\right)$

13) $f^{-1}(x) = -\sin^{-1}(x + 3) - 9$

14) $f^{-1}(x) = \frac{1}{6} \cos^{-1}\left(\frac{x}{9}\right)$

15) Domain of f : $(-\infty, \infty)$

Domain of f^{-1} : $[-2, 2]$

16) Domain of f : $(-\infty, \infty)$

Domain of f^{-1} : $[6, 8]$

17) Domain of f : $x \neq \frac{(2k+1)\pi}{2} + 5$; k an integer

Domain of f^{-1} : $(-\infty, \infty)$

18) $x = \frac{\sqrt{2}}{10}$

19) $x = \frac{\sqrt{3}}{2}$

20) -1

21) $\frac{\sqrt{2}}{2}$

22) 0

23) $\frac{\sqrt{3}}{2}$

24) 2

25) $\frac{2\pi}{7}$

26) $\frac{3\pi}{4}$

27) -0.14

Answer Key

Testname: UNTITLED1

28) 1.13

29) 1.14

30) $\frac{u\sqrt{u^2+1}}{u^2+1}$

31) $\frac{u\sqrt{u^2+1}}{u^2+1}$

32) $\{\pi\}$

33) $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

34) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

35) $\left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$

36) $\left\{\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}\right\}$

37) 45°

38) $\frac{2 \sin \theta + 1}{\sin \theta - 1}$

39) $-2 \cot^2 \theta$

40) $(\sec u - \tan u)(\sec u + \tan u) = \sec^2 u - \tan^2 u = 1$

41) $9 \csc^2 \theta - 6 \cot^2 \theta = 3 \csc^2 \theta + 6 \csc^2 \theta - 6 \cot^2 \theta = 3 \csc^2 \theta + 6 (\csc^2 \theta - \cot^2 \theta) = 3 \csc^2 \theta + 6$

42) $\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} = \frac{(\csc x + 1) \sin x + (\csc x - 1) \sin x}{\csc^2 x - 1} = \frac{1 + \sin x + 1 - \sin x}{\cot^2 x} = 2 \tan^2 x.$

43) $1 + \sec^2 x \sin^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x = \sec^2 x.$