PEARSON TEXAS ALGEBRA II

STUDENT TEXT AND HOMEWORK HELPER

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UNIT 1 ATTRIBUTES OF FUNCTIONS AND THEIR INVERSES

TOPIC 1 Functions

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS (TEKS) FOCUS

(2)(A) Graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.

(2)(B) Graph and write the inverse of a function using notation such as $f^{-1}(x)$.

(2)(C) Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range.

(2)(D) Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

(7)(B) Add, subtract, and multiply polynomials.

(7)(I) Write the domain and range of a function in interval notation, inequalities, and set notation.

TOPIC 2 Absolute Value Equations and Functions

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS (TEKS) FOCUS

(2)(A) Graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.

(6)(C) Analyze the effect on the graphs of $f(x) = |x|$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.

(6)(D) Formulate absolute value linear equations.

(6)(E) Solve absolute value linear equations.

(6)(F) Solve absolute value linear inequalities.

(7)(I) Write the domain and range of a function in interval notation, inequalities, and set notation.

Topic 1 | Functions

TOPIC OVERVIEW

VOCABULARY

English/Spanish Vocabulary Audio Online:

English	Spanish
asymptote, *p. 11*	asíntota
composite function, *p. 16*	función compuesta
dependent variable, *p. 4*	variable dependiente
domain, *p. 4*	dominio
function, *p. 4*	función
independent variable, *p. 4*	variable independiente
maximum, *p. 12*	máximo
minimum, *p. 12*	mínimo
parent function, *p. 11*	función elemental
range, *p. 4*	rango
relation, *p. 4*	relación
vertical-line test, *p. 4*	Prueba de la recta vertical

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3-Act Math

The Super Stairs

The staircase is one of the oldest architectural structures, dating to as early as 6000 B.C. These earliest staircases were often made of wood trunks fitted together and were built to make it easier for humans to navigate the terrain.

Today, "running stairs" is a popular exercise routine. Many people run stairs at stadiums or arenas. They run up one set of stairs, across the row of seats, and then down another set of stairs. Up and down, up and down. How many stairs do you think you could run up and down without stopping? Think about this as you watch this 3-Act Math video.

Scan page to see a video for this 3-Act Math Task.

If You Need Help...

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You'll find definitions of math terms in both English and Spanish. All of the terms have audio support.

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Refer to your notes and solutions in your Student Companion. Remember that your Student Companion is also available as an ACTIVebook accessible on any digital device.

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Not sure how to do some of the practice exercises? Check out the Virtual Nerd videos for stepped-out, multi-level instructional support.

1-1 Relations and Functions

TEKS FOCUS

TEKS (7)(I) Write the domain and range of a function in interval notation, inequalities, and set notation.

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple **representations**, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (1)(E)

VOCABULARY

- **Dependent variable** – The dependent variable, $f(x)$, represents the output of a function.
- **Domain** – the set of inputs, also called x-coordinates, of the ordered pairs
- **Function** – a relation in which each element of the domain corresponds with exactly one element of the range
- **Function notation** – The function notation $f(x)$ shows the function name f and also represents the range value $f(x)$ for the domain value x.
- **Function rule** – an equation that represents an output value in terms of an input value
- **Independent variable** – The independent variable, x, represents the input of the function.
- **Interval notation** – a notation used to describe an interval on a number line
- **Range** – the set of outputs, also called y-coordinates, of the ordered pairs
- **Relation** – a set of pairs of input and output values
- **Set-builder notation** – a notation used to describe the elements of a set
- **Vertical-line test** – The vertical-line test states if a vertical line passes through more than one point on the graph of a relation, then the relation is *not* a function.

- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

A pairing of items from two sets is special if each item from one set pairs with exactly one item from the second set.

take note — Key Concept — Four Ways to Represent Relations

Ordered Pairs

(input, output)

(x, y)
$(-3, 4)$
$(3, -1)$
$(4, -1)$
$(4, 3)$

Mapping Diagram

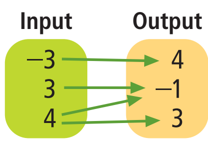
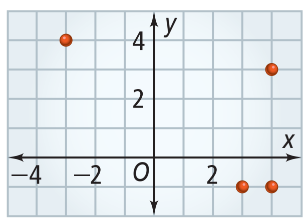


Table of Values

x Input	y Output
−3	4
3	−1
4	−1
4	3

Graph



take note

Key Concept Set-Builder Notation

Set-builder notation is another way to write sets. It describes the properties an element must have to be included in a set. For example, you can write the set $\{2, 4, 6, 8, \ldots\}$ in set-builder notation as $\{x \mid x \text{ is a natural number}, x \text{ is a multiple of } 2\}$. You read this as "the set of all natural numbers x, such that x is a multiple of 2."

Set-builder notation

Use a variable.

Describe the limits on the variable.

$$T = \{x \mid x \text{ is a natural number}, x < 6\}$$

Unless otherwise specified, you can assume that the set consists of real numbers. This way, if you want to write the set of all real numbers except 0, you can just write $\{x \mid x \neq 0\}$.

take note

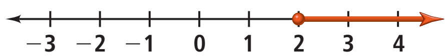
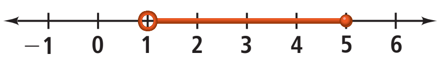
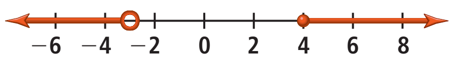
Key Concept Interval Notation

You can use an inequality such as $x \leq -3$ to describe a portion of the number line called an *interval*. You can also use *interval notation* to describe an interval on the number line. **Interval notation** includes the use of three special symbols. These symbols include

parentheses: Use (or) when a $<$ or $>$ symbol indicates that the interval's endpoints are *not* included.

brackets: Use [or] when a $\leq$ or $\geq$ symbol indicates that the interval's endpoints *are* included.

infinity: Use ∞ when the interval continues forever in a *positive* direction.
Use $-\infty$ when the interval continues forever in a *negative* direction.

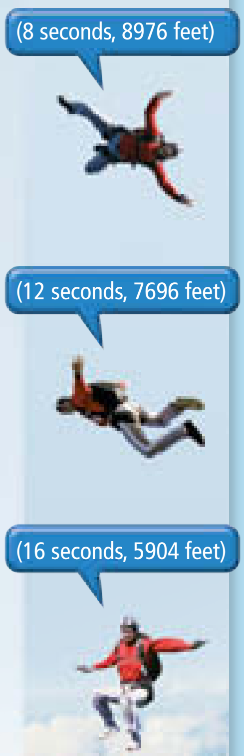
Inequality	Graph	Interval Notation
$x \geq 2$		$[2, \infty)$
$x < 2$		$(-\infty, 2)$
$1 < x \leq 5$		$(1, 5]$
$x < -3$ or $x \geq 4$		$(-\infty, -3)$ or $[4, \infty)$

Problem 1

TEKS Process Standard (1)(D)

Representing a Relation

Skydiving When skydivers jump out of an airplane, they experience free fall. The photos show various heights of a skydiver at different times during free fall, ignoring air resistance. How can you represent this relation in four different ways?



Think

What is the input? The output?
The input is the time. The output is the height above the ground.

Mapping Diagram

Input	Output
0 →	10,000
4 →	9744
8 →	8976
12 →	7696
16 →	5904

Ordered Pairs

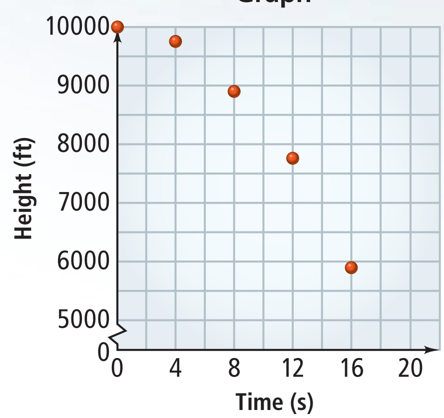
$\{(0, 10{,}000), (4, 9744), (8, 8976), (12, 7696), (16, 5904)\}$

Table of Values

Time (s)	Height (ft)
0	10,000
4	9744
8	8976
12	7696
16	5904

Each time value represents an input, which is paired with its corresponding output value (height).

Graph



Problem 2

Finding Domain and Range

Think

How could you use the mapping diagram in Problem 1 to find the domain and range?
The *input* corresponds to the domain of the relation. The *output* corresponds to the range.

Use the relation from Problem 1. What are the domain and range of the relation?

The relation is $\{(0, 10{,}000), (4, 9744), (8, 8976), (12, 7696), (16, 5904)\}$.

The domain is the set of x-coordinates. $\{0, 4, 8, 12, 16\}$

The range is the set of y-coordinates. $\{10{,}000, 9744, 8976, 7696, 5904\}$

Problem 3

TEKS Process Standard (1)(D)

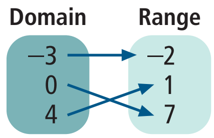
Identifying Functions

Plan

How can you use a mapping diagram to determine whether a relation is a function?
A function has only one arrow from each element of the domain.

Is the relation a function?

A



Each element in the domain corresponds with exactly one element in the range. This relation is a function.

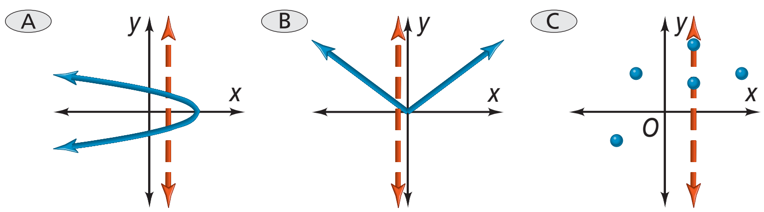
B $\{(4, -1), (8, 6), (1, -1), (6, 6), (4, 1)\}$

Each x-coordinate must correspond to only one y-coordinate. The x-coordinate 4 corresponds to -1 and 1. The relation is *not* a function.

Problem 4

Using the Vertical-Line Test

Use the vertical-line test. Which graph(s) represent functions?



Graphs A and C fail the vertical-line test because for each graph, a vertical line passes through more than one point. They do not represent functions. Graph B does not fail the vertical-line test, so it represents a function.

Think

Is a relation a function if it passes through the y-axis twice?
No; the y-axis is a vertical line, so the relation fails the vertical-line test.

Problem 5

TEKS Process Standard (1)(E)

Plan

How do you find the output?
Substitute the input into the function rule and simplify.

Using Function Notation

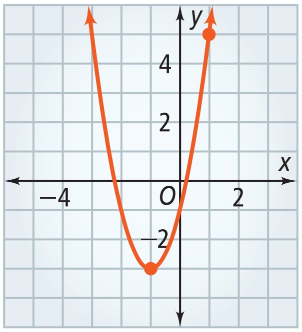
For $f(x) = -2x + 5$, what is the output for the inputs -3, 0, and $\frac{1}{4}$?

x Input	Function Rule $f(x) = -2x + 5$	$f(x)$ Output
-3	$f(-3) = -2(-3) + 5$	11
0	$f(0) = -2(0) + 5$	5
$\frac{1}{4}$	$f\left(\frac{1}{4}\right) = -2\left(\frac{1}{4}\right) + 5$	$4\frac{1}{2}$

Problem 6

Using Interval and Set Notation

A Use set notation to write the domain and range of the function $y = 2(x + 1)^2 - 3$ shown in the graph.



The domain is the collection of x-values for which the function is defined. The function $y = 2(x + 1)^2 - 3$ is defined for all real values of x. The function has a minimum value of -3 when $x = -1$, but generates all the real numbers greater than or equal to -3 as outputs of the function.

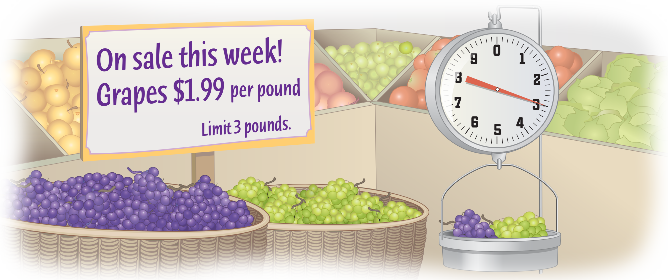
In set notation, the domain of the function is $\{x \mid x \text{ is a real number}\}$.

The range of the function is $\{y \mid y \geq -3\}$.

Think

How can you tell that the range variable cannot be less than negative 3?
The parabola opens up, and the vertex is point $(-1, -3)$. There are no y-values less than the y-value of the vertex.

B Use interval notation to write the domain and range of the function $C(x) = 1.99x$, where x is the weight of the grapes (pounds) and $C(x)$ is cost (dollars).



The domain variable describes the amount of grapes in pounds, which is at most 3 pounds. The range variable describes the cost for the grapes, which is at most $(1.99) \cdot 3 = 5.97$, or \$5.97.

In interval notation, the domain of the function is $[0, 3]$. The range of the function is $[0, 5.97]$.

PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to PearsonTEXAS.com.

Use Multiple Representations to Communicate Mathematical Ideas (1)(D) **Every year, the Rock and Roll Hall of Fame and Museum inducts legendary musicians and musical acts to the Hall. The table shows the number of inductees for each year.**

Rock and Roll Hall of Fame Inductees

Year	Number of Inductees	Year	Number of Inductees
2001	11	2004	8
2002	8	2005	7
2003	9	2006	6

Source: Rock and Roll Hall of Fame

1. Represent the data using each of the following:

a. a mapping diagram

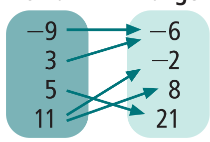
b. ordered pairs

c. a graph on the coordinate plane

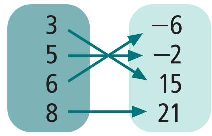
2. What are the domain and range of this relation?

Use Multiple Representations to Communicate Mathematical Ideas (1)(D)
For Exercises 3–9, determine whether each relation is a function. Explain.

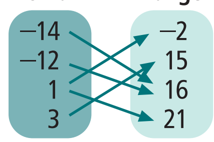
3. Domain Range



4. Domain Range

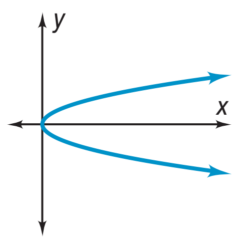


5. Domain Range

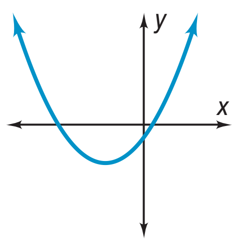


6. $\{(3, -9), (11, 21), (121, 34), (34, 1), (23, 45)\}$

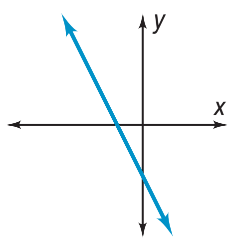
7.



8.



9.



STEM **10. Apply Mathematics (1)(A)** The time required for a certain chemical reaction is related to the amount of catalyst present during the reaction. The domain of the relation is the number of grains of catalyst, and the range is the number of seconds required for a fixed amount of the chemical to react. The table shows the data from several reactions.

a. Is the relation a function?

b. If the domain and range were interchanged, would the relation be a function? Explain.

Catalyst and Reaction Time

Number of Grains	Number of Seconds
2.0	180
2.5	6
2.7	0.05
2.9	0.001
3.0	6
3.1	15
3.2	37
3.3	176

Evaluate each function for the given value of x, and write the input x and output $f(x)$ as an ordered pair.

11. $f(x) = 17x + 3$ for $x = 4$

12. $f(x) = -\frac{2x+1}{3}$ for $x = -5$

13. The price of a beverage is a function of its size, as shown in the table. State the domain and range of the function in set notation.

Beverages

Size (ounces)	Price (dollars)
14	1.99
18	2.59
24	3.49

14. **Apply Mathematics (1)(A)** A cube is a solid figure with six square faces. If the edges of a cube have length 1.5 cm, what is the surface area of the cube?

15. **Apply Mathematics (1)(A)** Suppose you have a box with a 4×4-in. square base and variable height h. The surface area of this box is a function of its height. Write a function to represent the surface area. Evaluate the function for $h = 6.5$ in.

16. **Analyze Mathematical Relationships (1)(F)** Suppose a function pairs items from set A with items from set B. You can say that the function maps *into* set B. If the function uses every item from set B, the function maps *onto* set B. Does each function below map the set of whole numbers *into* or *onto* the set of whole numbers?
   a. Function f doubles every number.
   b. Function g maps every number to 1 more than that number.
   c. Function h maps every number to itself.
   d. Function j maps every number to its square.

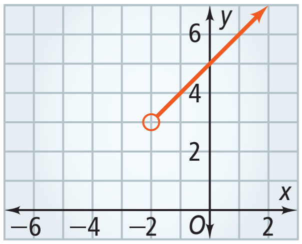
17. **Apply Mathematics (1)(A)** Given the functions $f(x) = 3x - 21$ and $g(x) = 3x + 21$, show that the function $f(x) - g(x)$ is a constant for all the values of x.

State the domain and range of each function using the specified notation.

18. $g(x) = -x^2$; set notation

19. $y = 3 - x$; interval notation

20. function in the graph shown right; set notation



TEXAS Test Practice

21. If $f(x) = -3x + 7$ and $g(x) = -7x + 3$, what is the value of $f(-3) - g(3)$?
   A. 40 **B.** 34 **C.** 8 **D.** −8

22. What is the formula for the volume of a cylinder, $V = \pi r^2 h$, solved for h?
   F. $h = \frac{r^2}{\pi V}$ **G.** $h = \frac{\pi V}{r^2}$ **H.** $h = \frac{V}{\pi r^2}$ **J.** $h = \frac{\pi r^2}{V}$

23. Which of the following statements are true?
   I. $-(-6) = 6$ and $-(-4) > -4$
   II. $-(-4) < 4$ or $-10 > 10 - 10$
   III. $5 + 6 = 11$ or $9 - 2 = 11$
   IV. $17 > 2$ or $6 < 9$
   A. I and II only
   B. I, II, and III only
   C. I, III, and IV only
   D. III and IV only

1-2 Attributes of Functions

TEKS FOCUS

TEKS (2)(A) Graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (7)(I)

VOCABULARY

- **Asymptote** – a line that a graph approaches as x or y increases in absolute value
- **Asymptotic behavior** – A graph that approaches an asymptote exhibits asymptotic behavior.
- **Cubic parent function** – the function $f(x) = x^3$
- **Maximum** – A maximum value of a function on an interval is the greatest y-value for the x-values in the interval.
- **Minimum** – A minimum value of a function on an interval is the smallest y-value for the x-values in the interval.
- **Parent function** – the simplest form in a set of functions that form a family
- **Reciprocal parent function** – the function $f(x) = \frac{1}{x}$
- **Symmetry** – A function has symmetry if a reflection or rotation carries its graph onto itself.
- ***x*-intercept** – a point at which a graph crosses the x-axis (or the x-coordinate of that point)
- ***y*-intercept** – a point at which a graph crosses the y-axis (or the y-coordinate of that point)

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

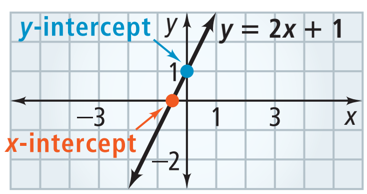
Functions are grouped into families with specific qualities. Understanding these qualities and key attributes helps you graph and work with various types of functions.

take note

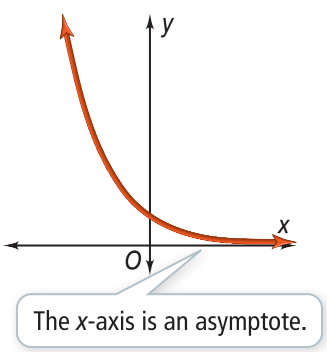
Key Concept Attributes of Functions

The point at which a line crosses the x-axis is an ***x*-intercept**. The x-intercept of $y = 2x + 1$ is $(-\frac{1}{2}, 0)$ or $-\frac{1}{2}$.

The point at which a line crosses the y-axis is a ***y*-intercept**. The y-intercept of $y = 2x + 1$ is $(0, 1)$ or 1.



An **asymptote** is a line that a graph approaches as x or y increases in absolute value. A graph that approaches an asymptote has **asymptotic behavior**.

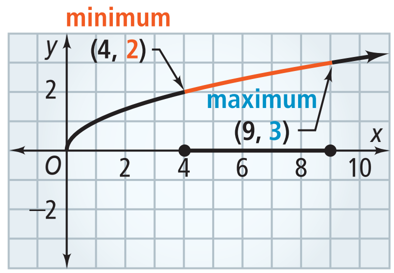


take note

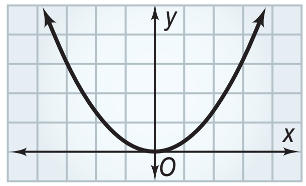
Key Concept Attributes of Functions *(cont.)*

A **maximum** value of a function on an interval is the greatest y-value for the x-values in the interval.

A **minimum** value of a function on an interval is the smallest y-value for the x-values in the interval.



A function has **symmetry** if a reflection or rotation carries its graph onto itself.



Problem 1

TEKS Process Standard (1)(E)

Graphing and Analyzing a Function

Plan

How do you get started?
Since the function is in slope-intercept form, you know the slope and y-intercept of the line represented by this function.

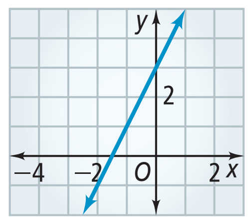
A **Graph the linear function $f(x) = 2x + 3$.**

The linear function has a slope of 2 and a y-intercept of (0, 3).

Start at (0, 3).

Move right 1 and up 2 to (1, 5).

Draw a line through the two points.



B **Find the domain, range, and intercepts of the function. Then find the minimum and maximum values of $f(x)$ on the interval $[-2, 5]$.**

Both the domain and range of $f(x)$ are all real numbers. Therefore, the domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

The y-intercept is (0, 3). To find the x-intercept, set $f(x)$ equal to 0.

$2x + 3 = 0$	Set $f(x) = 0$.
$x = -1.5$	Solve for x.

The x-intercept is $(-1.5, 0)$.

Since the entire graph of $f(x)$ slants upward, the minimum and maximum values occur at the endpoints of the interval. Therefore, $f(-2) = -1$ is the minimum value and $f(5) = 13$ is the maximum value on the interval $[-2, 5]$.

Problem 2

Analyzing Key Attributes of $f(x) = x^3$

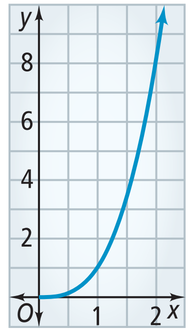
A store sells cube-shaped storage containers. The cubic parent function $f(x) = x^3$ gives the volume, in cubic inches, of a container with edge length x inches.

Think

How can you tell that the domain variable cannot be negative?
The domain represents the length of a container, which cannot be a negative value.

A **Graph the function $f(x)$. Determine the domain and range.**

Since the length of an edge cannot be negative, the domain is $[0, \infty)$. Since the cube of any positive number is also positive, the range is $[0, \infty)$.



B **Does the graph have any intercepts? If so, interpret their meaning in this context.**

The graph intersects the axes at the point $(0, 0)$. This is both the x-intercept and y-intercept of the graph. In this context, a cube with edge length 0 inches has a volume of 0 cubic inches.

Problem 3

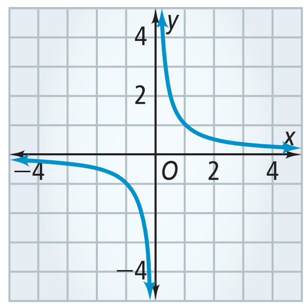
Analyzing Key Attributes of $f(x) = \frac{1}{x}$

Think

How do you know that the graph of $f(x)$ does not cross the x-axis or the y-axis?
If $x = 0$, the function is undefined. Also, there are no values of x that make $f(x) = 0$.

A **Graph the function $f(x) = \frac{1}{x}$. Make a table. Plot the points on the graph.**

x	-10	-1	-0.1	-0.01	0.01	0.1	1	10
$f(x)$	-0.1	-1	-10	-100	100	10	1	0.1



B **Analyze the graph to determine the asymptotic behavior, intercepts, and symmetry. State the domain and range in interval notation, inequalities, and set notation.**

The value of $f(x)$ approaches 0 as x increases in absolute value. Therefore $y = 0$ is a horizontal asymptote. The value of x approaches 0 as y increases in absolute value. Therefore $x = 0$ is a vertical asymptote.

The graph does not have any x- or y-intercepts.

The graph is symmetric with respect to the origin, and the lines $y = x$ and $y = -x$.

	Domain	**Range**
Interval notation	$(-\infty, 0)$ and $(0, \infty)$	$(-\infty, 0)$ and $(0, \infty)$
Inequalities	real numbers x such that $x < 0$ or $x > 0$	real numbers y such that $y < 0$ or $y > 0$
Set notation	$\{x \mid x \neq 0\}$	$\{y \mid y \neq 0\}$

C **Find the maximum and minimum values of $f(x) = \frac{1}{x}$ on the interval $[0.5, 4]$.**

Because the y-value decreases on the interval $[0.5, 4]$ and $f(0.5) = 2$, the maximum value on the interval is 2. The minimum value on the interval $[0.5, 4]$ is 0.25 since $f(4) = 0.25$.

PRACTICE and APPLICATION EXERCISES

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For each linear function, find the domain, range, and intercepts. Then find the minimum and maximum values of $f(x)$ on the given interval. Write the domain and range in interval notation.

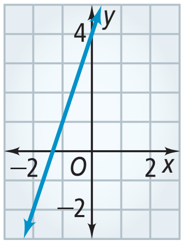
1. $f(x) = -x$; $[3, 10]$

2. $f(x) = 3x + 11$; $[-5, 0]$

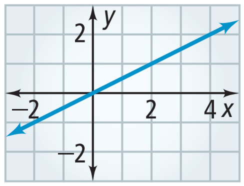
3. $f(x) = \frac{2 - x}{3}$; $[-8, 8]$

4. $f(x) = -12$; $[0, 20]$

5. $[-1, 0]$



6. $[-8.5, -6]$



7. The linear function $d(t) = 4 - 3t$ gives your distance d (in miles) from home, t hours after you leave the library.

a. Sketch a graph of the function. Write the domain and range in inequalities for $d(t)$ in this context.

b. What are the intercepts of the graph? Explain the meaning of any intercepts in this context.

8. Sketch a graph of the cubic parent function $f(x) = x^3$. Use the graph to determine the intercepts, and write the domain and range in interval notation.

9. **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** A classmate graphs the function $f(x) = x^3 + 1$. He describes the graph as similar to the graph of $f(x) = x^3$, but shifted up 1 unit. For each attribute listed, describe any change from the cubic parent function to your classmate's function.

a. domain

b. range

c. x-intercept

d. y-intercept

A teacher draws a graph of $f(x) = x^3$ on the blackboard. The students then make the statements below about the graph. Determine if the statement is true or false.

10. The graph does not have any minimum or maximum.

11. The graph has one x-intercept, but no y-intercept.

12. The range of the function is $(-\infty, \infty)$.

13. The graph is a reflection of itself over the y-axis.

14. **Explain Mathematical Ideas (1)(G)** If $f(-x) = -f(x)$ for all values of x in the domain of $f(x)$, then the graph of $f(x)$ has point symmetry about the origin. Do the parent functions $f(x) = x^2$ or $f(x) = x^3$ have this symmetry? Explain.

Graph the function $g(x) = \frac{2}{x}$ by making a table of values and plotting points. How does the graph compare to its parent function, $f(x) = \frac{1}{x}$? Describe any change for each of the following.

15. Domain
16. Range
17. Asymptotic behavior
18. Intercepts
19. Symmetry
20. Minimum value on the interval $[1, 8]$.

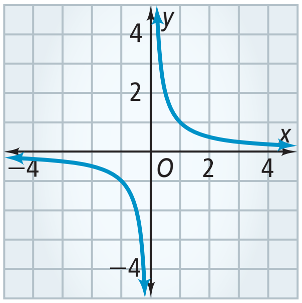
Match each function with a corresponding attribute so that you use each of the attributes exactly once.

21. $y = x^2$
22. $y = -6$
23. $y = x^3$
24. $y = \frac{1}{x}$
25. $y = -2x + 5$

A. Exactly one x-intercept
B. Asymptote $x = 0$
C. Exactly one y-intercept
D. Symmetry about the origin
E. a minimum

TEXAS Test Practice

26. Which function is shown in the graph?



A. $y = \frac{1}{x}$
B. $y = -\frac{1}{x}$
C. $y = x^3$
D. $y = -x$

27. Which best describes all of the intercepts of the graph of $f(x) = x^3$?

F. $(0, 0)$ is the x-intercept.
G. $(0, 0)$ is the y-intercept.
H. $(0, 0)$ is the x-intercept and the y-intercept.
J. The function has no intercepts.

28. For the domain $x < -1$, which of the following is the range of $f(x) = x^3$?

A. $y < -1$
B. $y > -1$
C. $y < 1$
D. $y > 1$

29. Explain why the function $f(x) = \frac{1}{x}$ has no maximum value on the interval $(0, 10]$. Does it have a minimum value on this interval?

1-3 Function Operations and Composition

TEKS FOCUS

TEKS (7)(B) Add, subtract, and multiply polynomials.

TEKS (1)(A) **Apply** mathematics to problems arising in everyday life, society, and the workplace.

Additional TEKS (1)(E)

VOCABULARY

- **Composite function** – A composite function is a combination of two functions such that the output from the first function becomes the input for the second function.
- **Apply** – use knowledge or information for a specific purpose, such as solving a problem

ESSENTIAL UNDERSTANDING

You can add, subtract, multiply, and divide functions based on how you perform these operations for real numbers. One difference, however, is that you must consider the domain of each function.

take note

Key Concepts Function Operations

Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of both f and g. Also, the domain of the quotient function does not contain any x-value for which $g(x) = 0$.

Key Concept Composition of Functions

The composition of function g with function f is written as $g \circ f$ and is defined as $(g \circ f)(x) = g(f(x))$. The domain of $g \circ f$ consists of the x-values in the domain of f for which $f(x)$ is in the domain of g.

$$(g \circ f)(x) = \underbrace{g(\underbrace{f(x)}_{1})}_{2}$$

1. Evaluate $f(x)$ first.

2. Then use $f(x)$ as the input for g.

Function composition is not commutative since $f(g(x))$ does not always equal $g(f(x))$.

Problem 1

TEKS Process Standard (1)(E)

Adding and Subtracting Functions

Think

How can you use the Distributive Property to subtract the expression $(12 + x)$?
Distribute the negative sign to both terms in the expression.

Let $f(x) = 4x + 7$ and $g(x) = 12 + x$. What are $f + g$ and $f - g$? What are their domains?

$$(f + g)(x) = f(x) + g(x) = (4x + 7) + (12 + x) = 5x + 19$$

$$(f - g)(x) = f(x) - g(x) = (4x + 7) - (12 + x) = 3x - 5$$

The domain of both f and g is the set of all real numbers. The domain of both $f + g$ and $f - g$ is the set of numbers common to the domain of both f and g, which is the set of all real numbers.

Problem 2

Multiplying and Dividing Functions

Let $f(x) = x^2 - 9$ and $g(x) = x + 3$. What are $f \cdot g$ and $\frac{f}{g}$ and their domains?

$$(f \cdot g)(x) = f(x) \cdot g(x) = (x^2 - 9)(x + 3)$$

$$= x^3 + 3x^2 - 9x - 27$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 9}{x + 3} = \frac{(x - 3)(x + 3)}{x + 3} = x - 3,\ x \neq -3$$

Think

Is the domain of $\frac{f}{g}$ the domain of $x - 3$?
No; the fraction can only be simplified and the function is only defined when $g(x) \neq 0$.

The domain of both f and g is the set of real numbers, so the domain of $f \cdot g$ is also the set of real numbers.

The domain of $\frac{f}{g}$ is the set of all real numbers except $x \neq -3$, because $g(-3) = 0$. The definition of $\frac{f}{g}$ requires that you consider the zero denominator in the *original* expression for $\frac{f(x)}{g(x)}$ despite the fact that the simplified form has the domain of all real numbers.

Problem 3

Composing Functions

Let $f(x) = x - 5$ and $g(x) = x^2$. What is $(g \circ f)(-3)$?

Think

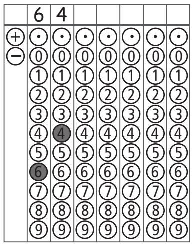
Which function is substituted into the other?
Use $f(x)$ as the input for g.

Method 1

$$(g \circ f)(x) = g(f(x))$$
$$= g(x - 5)$$
$$= (x - 5)^2$$
$$(g \circ f)(-3) = (-3 - 5)^2$$
$$= (-8)^2$$
$$= 64$$

Method 2

$$(g \circ f)(-3) = g(f(-3))$$
$$= g(-3 - 5)$$
$$= g(-8)$$
$$= (-8)^2$$
$$= 64$$



Problem 4

TEKS Process Standard (1)(A)

Using Composite Functions

You have a coupon good for \$5 off the price of any large pizza. You also get a 10% discount on any pizza if you show your student ID. How much more would you pay for a large pizza if the cashier applies the coupon first?

Know	Need	Plan
The coupon value and the discount rate	The difference between the results of applying the discount or coupon first	• Compose two functions in two ways. • Then find the difference in their results.

Step 1 Find functions C and D that model the cost of a large pizza.

Let $x =$ the price of a large pizza.

Cost using the coupon: $C(x) = x - 5$

Cost using the 10% discount: $D(x) = x - 0.1x = 0.9x$

Step 2 Compose the functions to apply the discount and then the coupon.

$(C \circ D)(x) = C(D(x))$ Apply the discount, $D(x)$, first.

$= C(0.9x) = 0.9x - 5$

Step 3 Compose the functions to apply the coupon and then the discount.

$(D \circ C)(x) = D(C(x))$ Apply the coupon, $C(x)$, first.

$= D(x - 5) = 0.9(x - 5) = 0.9x - 4.5$

Step 4 Subtract the functions to find how much more you would pay if the cashier applies the coupon first.

$$(D \circ C)(x) - (C \circ D)(x) = (0.9x - 4.5) - (0.9x - 5)$$
$$= -4.5 + 5$$
$$= 0.5$$

You pay \$.50 more if the cashier applies the coupon first.

PRACTICE and APPLICATION EXERCISES

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Let $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$. Perform each function operation and then find the domain.

1. $(f \cdot g)(x)$ **2.** $(f - g)(x)$ **3.** $\frac{g}{f}(x)$

Let $g(x) = 2x$ and $h(x) = x^2 + 4$. Find each value or expression.

4. $(h \circ g)(-5)$ **5.** $(g \circ h)(0)$ **6.** $(g \circ h)(a)$

7. Apply Mathematics (1)(A) A computer store offers a 5% discount off the list price x for any computer bought with cash, rather than put on credit. At the same time, the manufacturer offers a \$200 rebate for each purchase of a computer.

a. Write a function $f(x)$ to represent the price after the cash discount.

b. Write a function $g(x)$ to represent the price after the \$200 rebate.

c. Suppose the list price of a computer is \$1500. Use a composite function to find the price of the computer if the discount is applied before the rebate.

d. Suppose the list price of a computer is \$1500. Use a composite function to find the price of the computer if the rebate is applied before the discount.

8. Apply Mathematics (1)(A) Suppose the function $f(x) = 0.15x$ represents the number of U.S. dollars equivalent to x Chinese yuan and the function $g(y) = 14.07y$ represents the number of Mexican pesos equivalent to y U.S. dollars.

a. Write a composite function that represents the number of Mexican pesos equivalent to x Chinese yuan.

b. Find the value in Mexican pesos of an item that costs 15 Chinese yuan.

Let $f(x) = 2x + 5$ and $g(x) = x^2 - 3x + 2$. Perform each function operation and then find the domain.

9. $-f(x) + 4g(x)$ **10.** $f(x) - 2g(x)$ **11.** $f(x) \cdot g(x)$

12. $-3f(x) \cdot g(x)$ **13.** $\dfrac{f(x)}{g(x)}$ **14.** $\dfrac{5f(x)}{g(x)}$

15. Apply Mathematics (1)(A) A craftsman makes and sells violins. The function $I(x) = 5995x$ represents the income in dollars from selling x violins. The function $P(y) = y - 100{,}000$ represents his profit in dollars if he makes an income of y dollars. What is the profit from selling 30 violins?

16. Suppose your teacher offers to give the whole class a bonus if everyone passes the next math test. The teacher says she will give everyone a 10-point bonus and increase everyone's grade by 9% of their score.

a. You earned a 75 on the test. Would you rather have the 10-point bonus first and then the 9% increase, or the 9% increase first and then the 10-point bonus?

b. Explain Mathematical Ideas (1)(G) Is this the best plan for all students? Explain.

Let $f(x) = 2 - x$ and $g(x) = \frac{1}{x}$. **Perform each function operation and then find the domain of the result.**

17. $(f + g)(x)$ **18.** $(f - g)(x)$ **19.** $(f \cdot g)(x)$

Let $f(x) = x^2$ **and** $g(x) = x - 3$. **Find each value or expression.**

20. $(f \circ g)(0)$ **21.** $(f \circ g)(-2)$ **22.** $(g \circ f)(3.5)$

23. Apply Mathematics (1)(A) A salesperson earns a 3% bonus on weekly sales over \$5000. Consider the following functions.

$g(x) = 0.03x$ $h(x) = x - 5000$

a. Explain what each function above represents.

b. Which composition, $(h \circ g)(x)$ or $(g \circ h)(x)$, represents the weekly bonus? Explain.

Let $g(x) = 3x + 2$ **and** $f(x) = \frac{x-2}{3}$. **Find each value.**

24. $f(g(1))$ **25.** $g(f(-4))$ **26.** $(f \circ g)(-2)$

27. Apply Mathematics (1)(A) Write a function rule that approximates each value.

a. The amount you save is a percent of what you earn. (You choose the percent.)

b. The amount you earn depends on how many hours you work. (You choose the hourly wage.)

c. Write and simplify a composite function that expresses your savings as a function of the number of hours you work. Interpret your results.

TEXAS Test Practice

28. Let $f(x) = x + 5$ and $g(x) = x^2 - 25$. What is the domain of $\frac{f}{g}(x)$?

A. All real numbers **C.** All real numbers except -5

B. All real numbers except 5 **D.** All real numbers except -5 and 5

29. Let $g(x) = x - 3$ and $h(x) = x^2 + 6$. What is $(h \circ g)(1)$?

F. -14 **G.** 4 **H.** 10 **J.** 15

30. Which number is a solution of $|3 - 2x| < 5$?

A. -6 **B.** -1 **C.** 2 **D.** 4

31. What is the coefficient of the x^3y^4 term in the expansion of $(3x - y)^7$? Show your work.

1-4 Inverse Functions

TEKS FOCUS

TEKS (2)(B) Graph and write the inverse of a function using notation such as $f^{-1}(x)$.

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple **representations**, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (2)(C), (2)(D), (7)(I)

VOCABULARY

- **Inverse function** – If function f pairs a value b with a, then its inverse, denoted f^{-1}, pairs the value a with b. If f^{-1} is also a function, then f and f^{-1} are inverse functions.
- **Inverse relation** – If a relation pairs element a of its domain with element b of its range, then the inverse relation "undoes" the relation and pairs b with a.
- **One-to-one function** – A one-to-one function is a function for which each y-value in the range corresponds to exactly one x-value in the domain.
- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

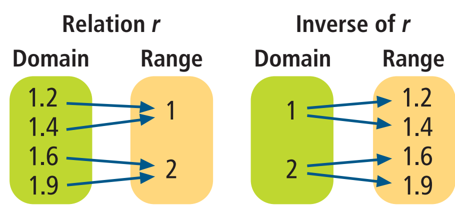
ESSENTIAL UNDERSTANDING

The inverse of a function may or may not be a function.

take note

Key Concept Inverse Relation

This diagram shows a relation r (a function) and its inverse (not a function). The range of the relation is the domain of the inverse. The domain of the relation is the range of the inverse.



Key Concept The Inverse of f

The inverse of a function f is denoted by f^{-1}. You read f^{-1} as "the inverse of f" or as "f inverse." The notation $f(x)$ is used for functions, but the relation f^{-1} may not even be a function.

Key Concept Composition of Inverse Functions

f and f^{-1} are inverse functions if and only if $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for x in the domains of f and f^{-1}, respectively.

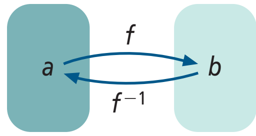
Key Concept One-to-One Functions and Their Inverses

take note

You know that for any function f, each x-value in the domain corresponds to one y-value in the range. For a **one-to-one function**, it is also true that each y-value in the range corresponds to exactly one x-value in the domain.

A one-to-one function f has an inverse f^{-1} that is also a function. If a function is not one-to-one, then its inverse is not a function.

Domain of f Range of f^{-1} | Range of f Domain of f^{-1}



Problem 1

TEKS Process Standard (1)(D)

Finding the Inverse of a Relation

A What is the inverse of relation s?

Relation s

x	y
0	−1
2	0
3	2
4	3

Switch the x and y values to get the inverse.

Inverse of Relation s

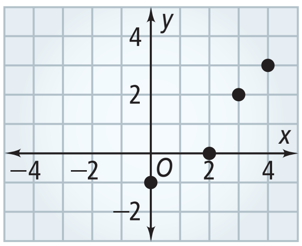
x	y
−1	0
0	2
2	3
3	4

B What are the graphs of s and its inverse?

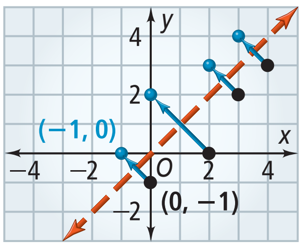
Think

(0, −1) is in s. How do you find the corresponding pair in the inverse of s?
Switch the coordinates. (−1, 0) is in the inverse of s.

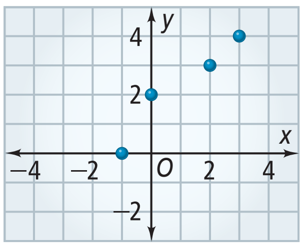
Relation s



Reversing the Ordered Pairs



Inverse of s



The graphs of a relation and its inverse are reflections of each other in the line $y = x$.

Problem 2

TEKS Process Standard (1)(D)

Finding an Equation for the Inverse

Consider the function described by $f(x) = -\frac{4}{3}x + 4$.

A Write the inverse of the function.

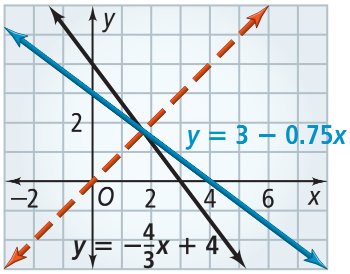
If you describe a relation or function by an equation in x and y, you can switch x and y to get an equation for the inverse.

$y = -\frac{4}{3}x + 4$	Replace $f(x)$ with y in the original function.
$x = -\frac{4}{3}y + 4$	Switch x and y.
$x - 4 = -\frac{4}{3}y$	Subtract 4 from both sides.
$-\frac{3}{4}x + 3 = y$	Multiply both sides by $-\frac{3}{4}$.
$-\frac{3}{4}x + 3 = f^{-1}(x)$	Replace y with $f^{-1}(x)$.

Think

Why do you solve for y?
If you solve for y, you can use it to easily generate the ordered pairs that are part of the inverse relation.

B Graph the function and its inverse on the same coordinate plane.



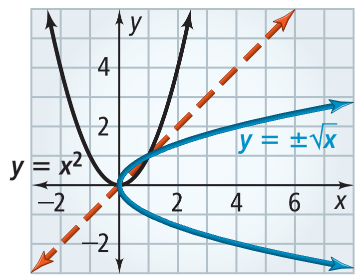
Problem 3

TEKS Process Standard (1)(F)

Graphing a Function and Its Inverse

A What are the graphs of $f(x) = x^2$ and its inverse, $f^{-1}(x) = \pm\sqrt{x}$?

The graph of $y = x^2$ is a parabola that opens upward with vertex (0, 0). The graph of the inverse is the reflection of the parabola across the line $y = x$.



B Is $f(x) = x^2$ one-to-one?

No, $f(x) = x^2$ is not a one-to-one function because there are y-values in the range of f that correspond to two different x-values in the domain of f.

C Analyze the relationship between $f(x)$ and $f^{-1}(x)$. What restrictions on the domain of $f(x)$ make $f^{-1}(x)$ a function?

In order for $f^{-1}(x)$ to be a function, there must be exactly one value in the range of $f^{-1}(x)$ for each value in the domain. To make this happen, instances where two distinct x-values in the domain of $f(x)$ are mapped to the same member of the range must be eliminated.

One way of producing this result is to restrict the domain of $f(x)$ to $x \geq 0$.

Think

Is there only one way to restrict the domain of $f(x)$ to make $f^{-1}(x)$ a function?
No, there are an infinite number of domains on which $f(x)$ is one-to-one, including the domains $\{x \mid x \leq 0\}$, $\{x \mid 7 < x < 11\}$, and $\{-10.5, -9.7, 3\}$.

Problem 4

TEKS Process Standard (1)(A)

Finding the Inverse When Domain and Range Have Restrictions

A grocery store pays managers \$20.00 per hour. The produce department manager is available to work a maximum of 50 hours per week.

A **Write the function, $f(x)$, that gives the wages the produce manager will earn by working x hours in a week.**

$f(x) = 20x$

B **What are the domain and range of f?**

The number of hours worked cannot be negative, and the manager is not available to work more than 50 hours in a week. Therefore, the domain of f is $\{x \mid 0 \le x \le 50\}$. The wages earned is the product of 20 and the number of hours worked, so the set $\{y \mid 0 \le y \le 1000\}$ makes up the range.

C **Find f^{-1}, the inverse of f.**

$y = 20x$	Original function
$x = 20y$	Switch x and y.
$0.05x = y$	Divide both sides by 20.

Think

How could a graph help you check your answer?
You could graph f^{-1} and see whether the graph passes the vertical-line test. If it does, f^{-1} is a function.

D **What are the domain and range of f^{-1}?**

The range of f is the domain of f^{-1}, which is the set of numbers between and including 0 and 1000. Since $y = 0.05x$, the range of f^{-1} is the set of numbers equal to 0.05 the elements of the domain: $\{y \mid 0 \le y \le 50\}$. Notice that the range of f^{-1} is the domain of f.

E **Is f^{-1} a function? Explain.**

For each x in the domain $\{x \mid 0 \le x \le 1000\}$, there is only one y-value in the range. So, $f^{-1} = 0.05x$, $0 \le x \le 1000$, is a function.

Problem 5

Composing Inverse Functions

Think

Are there any restrictions on the domains of $f(x)$ and $g(x)$ that will restrict the range of the functions?

There are no restrictions on the domain of $f(x)$ and $g(x)$. The domain for both functions is the set of all real numbers.

Use composition of functions to determine whether each pair of functions are inverse functions.

A **Let $f(x) = \frac{1}{2}x - 3$ and $g(x) = 2x + 6$.**

$$(f \circ g)(x) = f(g(x)) \qquad (g \circ f)(x) = g(f(x))$$

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(2x+6) \\ &= \tfrac{1}{2}(2x+6) - 3 \\ &= x + 3 - 3 \\ &= x \end{aligned} \qquad \begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g\left(\tfrac{1}{2}x - 3\right) \\ &= 2\left(\tfrac{1}{2}x - 3\right) + 6 \\ &= x - 6 + 6 \\ &= x \end{aligned}$$

The functions f and g are inverse functions.

B **Let $f(x) = 3x - 2$ and $g(x) = \frac{1}{3}x + 2$.**

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f\left(\tfrac{1}{3}x + 2\right) \\ &= 3\left(\tfrac{1}{3}x + 2\right) - 2 \\ &= x + 6 - 2 \\ &= x + 4 \end{aligned} \qquad \begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g(3x - 2) \\ &= \tfrac{1}{3}(3x-2) + 2 \\ &= x - \tfrac{2}{3} + 2 \\ &= x + 1\tfrac{1}{3} \end{aligned}$$

The functions f and g are not inverse functions.

PRACTICE and APPLICATION EXERCISES

ONLINE HOMEWORK

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to **PearsonTEXAS.com**.

Find the inverse, r, of each relation s.

1. $s = \{(1, 2), (1, 4), (3, 5), (7, 5), (9, 6)\}$

2. $s = \{(-7, 3), (-6, 4), (-4, 5), (-2, 7), (1, 10)\}$

3.

x	y
0	4
3	−3
6	2
9	5

4.

x	y
−5	3
−3	2
−2	4
−1	2

5. Graph the inverse of relation t from Exercise 4. Is the relation a function?

For each function, $f(x)$, write the equation of the inverse, $f^{-1}(x)$. Then graph the inverse on a coordinate plane.

6. $f(x) = \frac{1 - 3x}{2}$

7. $f(x) = 2x - 4$

8. $f(x) = x^2 - 1$

9. $f(x) = x + 3$

10. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

a. Find the inverse of the formula. Is the inverse a function?

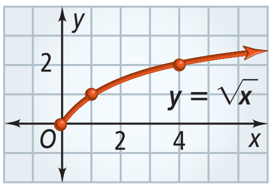
b. Use the inverse to find the radius of a sphere with a volume of 300 cubic inches.

For each function, $f(x)$, state the domain and range of the function. Then state the domain and range of the inverse, $f^{-1}(x)$, using set notation.

11.

x	$f(x)$
1	−3
2	−1
3	1
4	3

12.



13. $f(x) = 4 - x^2$

14. $f(x) = 2x - 1$

15. **Explain Mathematical Ideas (1)(G)** Explain why $f(x) = 9 - x^2$ is one-to-one on the domain $\{x \mid x \geq 3\}$ but $f(x)$ is not one-to-one on the domain $\{x \mid x$ is a real number$\}$.

For each function, find the equation of the inverse, $f^{-1}(x)$, and state the domain and range of $f^{-1}(x)$ using set notation. Then determine whether $f^{-1}(x)$ is a function.

16. $f(x) = \sqrt{x^2 - 9}$

17. $f(x) = \frac{3}{x}$

18. $f(x) = x^2 - 2$

19. $f(x) = \frac{2}{5}x - 4$

20. Explain Mathematical Ideas (1)(G) The domain of the function $f(x)$ contains 3 elements. What is the smallest possible quantity of elements in the range of the function? What is the greatest possible quantity of elements in the range of the function? What are the least and greatest possible number of elements in the range of $f(x)$ if the inverse, $f^{-1}(x)$, is also a function? Explain.

21. Explain Mathematical Ideas (1)(G) Explain how you can find the range of the inverse of $f(x) = \sqrt{3x + 5}$ without finding the inverse first.

Let $f(x) = 2x - 3$. Find each value.

22. $(f \circ f^{-1})(5)$

23. $(f^{-1} \circ f)(0.8)$

Use composition of functions to determine whether f and g are inverse functions.

24. $f(x) = x^2 + 25, g(x) = x - 5$

25. $f(x) = \frac{x}{3} - 2, g(x) = 3x + 6$

TEXAS Test Practice

26. Analyze Mathematical Relationships (1)(F) Let $f(x) = x^2 - 16$.

a. What are the domain and range of $f(x)$?

b. What are the domain and range of $f^{-1}(x)$?

c. Is $f(x)$ one-to-one? Explain your answer.

d. Give a restricted domain of $f(x)$, if one exists, so that $f^{-1}(x)$ a function.

27. Let $f(x) = x^3 - 8$. What is $f^{-1}(x)$?

A. $f^{-1}(x) = \sqrt[3]{x} + 8$

B. $f^{-1}(x) = \sqrt[3]{x + 8}$

C. $f^{-1}(x) = \sqrt[3]{x} + 2$

D. $f^{-1}(x) = \sqrt[3]{x + 2}$

28. Let $f(x) = \sqrt{2x - 6}$. What is the domain of $f^{-1}(x)$?

F. $\{x \mid x \geq 0\}$

G. $\{x \mid x \geq 3\}$

H. $\{x \mid x \geq 6\}$

J. $\{x \mid x \geq 9\}$

29. The graph of $f(x)$ lies entirely in quadrant II. In which quadrant(s) will the graph of $f^{-1}(x)$ lie? Explain your answer.

Topic 1 Review

TOPIC VOCABULARY

- asymptote, *p. 11*
- asymptotic behavior, *p. 11*
- composite function, *p. 16*
- cubic parent function, *p. 11*
- dependent variable, *p. 4*
- domain, *p. 4*
- function, *p. 4*
- function notation, *p. 4*
- function rule, *p. 4*
- independent variable, *p. 4*
- interval notation, *p. 5*
- inverse function, *p. 21*
- inverse relation, *p. 21*
- maximum, *p. 12*
- minimum, *p. 12*
- one-to-one function, *p. 22*
- parent function, *p. 11*
- range, *p. 4*
- reciprocal parent function, *p. 11*
- relation, *p. 4*
- set-builder notation, *p. 5*
- symmetry, *p. 12*
- vertex, *p. 4*
- vertical-line test, *p. 4*
- *x*-intercept, *p. 11*
- *y*-intercept, *p. 11*

Check Your Understanding

Choose the correct term to complete each sentence.

1. The (domain/range) of the function $y = x^2$ is $y \geq 0$.

2. A researcher studies the impact of a drug on cancer. The (dependent variable/independent variable) is the administration of the drug.

3. The function $4y - x = -20$ has an (x-intercept/y-intercept) of -5.

1-1 Relations and Functions

Quick Review

A **relation** is a set of ordered pairs. The **domain** of a relation is the set of x-coordinates. The **range** is the set of y-coordinates. When each element of the domain is paired with exactly one element of the range, the relation is a **function**.

Example

Determine whether the relation is a function. Find the domain and range.

$\{(5, 0), (8, 1), (1, 3), (5, 2), (3, 8)\}$

In this relation, the x-coordinate 5 is paired with both 0 and 2. This relation is not a function.

The domain is the set of x-coordinates, which is $\{5, 8, 1, 3\}$.

The range is the set of y-coordinates, which is $\{0, 1, 3, 2, 8\}$.

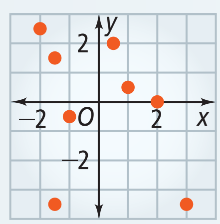
Exercises

Determine whether each relation is a function. Find the domain and range.

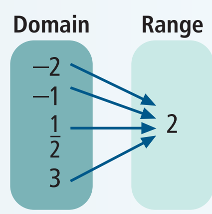
4. $\{(10, 2), (-10, 2), (6, 4), (5, 3), (-6, 7)\}$

5. $\{(4, 5), (1, 5), (3, 8), (4, 6), (10, 12)\}$

6.



7.



For each function, find $f(-2)$, $f(-0.5)$, and $f(3)$.

8. $f(x) = -x + 4$

9. $f(x) = \frac{3}{8}x - 3$

1-2 Attributes of Functions

Quick Review

An **asymptote** is a line that a graph approaches as x or y increases in absolute value.

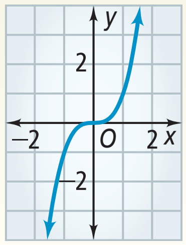
You can find the ***x*-intercept** of $y = -3x + 7$ by setting $y = 0$ and solving the resulting equation.

$0 = -3x + 12$	Set $y = 0$.
$x = 4$	Solve.

So, the x-intercept is $(4, 0)$ or 4.

Example

What is the range of the cubic parent function $f(x) = x^3$?



The range is the set of all outputs, or y-coordinates, of the ordered pairs of the function. Based on the graph, the range is all real numbers, or the interval $(-\infty, \infty)$.

Exercises

Find the domain, range, and intercepts of each function.

10. $x + 3y = -1$

11. $f(x) = 2x^3$

12. $f(x) = -x^2$

13. $y = \frac{1}{x}$

Find the minimum and maximum values of $f(x)$ on the given interval, if each value exists.

14. $f(x) = \frac{2}{3}; [-\infty, \infty]$

15. $f(x) = x^3; [2, 23]$

16. $f(x) = \frac{x-5}{2}; [-3, 0]$

17. $f(x) = \frac{1}{x}; [-\infty, -1]$

18. A classmate says that $f(x) = -x^3$ has a range of $y \leq 0$. Is your classmate correct? Explain.

19. Give an example of a function with a graph that has symmetry. Explain.

1-3 Function Operations and Composition

Quick Review

When performing function operations, you can use the same rules you used for real numbers, but you must take into consideration the domain and range of each function. The composition of function g with function f is defined as $(g \circ f)(x) = g(f(x))$.

Example

Let $f(x) = x + 3$ and $g(x) = x^2 - 2$. What is $(g \circ f)(-2)$?

$g(f(-2)) = g((-2) + 3)$	Evaluate $f(-2)$.
$= g(1)$	Simplify.
$= (1)^2 - 2$	Evaluate $g(f(-2))$.
$= -1$	Simplify.

Therefore, $(g \circ f)(-2) = -1$.

Exercises

Let $f(x) = x - 4$ and $g(x) = x^2 - 16$. Perform each function operation and then find the domain.

20. $f(x) + g(x)$

21. $g(x) - f(x)$

22. $f(x) \cdot g(x)$

23. $\frac{g(x)}{f(x)}$

Let $g(x) = 5x - 2$ and $h(x) = x^2 + 1$. Find the value of each expression.

24. $(h \circ g)(-1)$

25. $(h \circ g)(0)$

26. $(g \circ h)(2)$

27. $(g \circ h)(a)$

28. A grocery store is offering a 50% discount off a \$4.00 box of cereal. You also have a \$1.00 off coupon for the same cereal. Use a composite function to show whether it is better to use the coupon before or after the store discount.

1-4 Inverse Functions

Quick Review

An **inverse function**, f^{-1}, maps every element of the range of a function, f, to the member of the domain that generated that value.

If $f(a) = b$, then $f^{-1}(b) = a$ for all a in the domain of f and all b in the range of f.

Example

Let $f(x) = 4x - 8$. Find $f^{-1}(x)$.

Replace $f(x)$ with y, switch x and y, then solve for y to find the inverse equation.

$y = 4x - 8$	Replace $f(x)$ with y.
$x = 4y - 8$	Switch x and y.
$x + 8 = 4y$	Add 8 to both sides.
$\frac{x}{4} + 2 = y$	Divide both sides by 4.
$\frac{x}{4} + 2 = f^{-1}(x).$	Replace y with $f^{-1}(x)$.

When $f(x) = 4x - 8$, $f^{-1}(x) = \frac{x}{4} + 2$.

Exercises

For each function, f, find f^{-1}.

29. $f(x) = 7 - 3x$

30. $f(x) = x^2 - 16$

For each function f, find f^{-1} and state the domain and range of the inverse using set notation.

31. $f(x) = \sqrt{x - 9}$

32. $f(x) = \frac{2}{3}x + 6$

Let $f(x) = 13x + 2$. Find each value.

33. $(f \circ f^{-1})(-6)$

34. $(f^{-1} \circ f)(0.03)$

35. Consider the function defined in the table.

Function s

x	y
0	−1
2	0
3	2
4	3

a. Write the inverse of function s as a set of ordered pairs.

b. Graph function s and its inverse on separate coordinate grids.

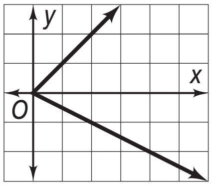
c. Is s a function? Is s^{-1} a function? Explain your answer.

Multiple Choice

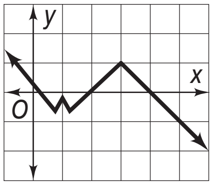
Read each question. Then write the letter of the correct answer on your paper.

1. Which relation is a function?

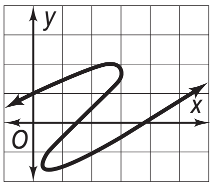
A.



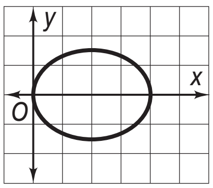
B.



C.



D.



2. For $f(x) = 2x - 3$ find $f\left(-\frac{1}{4}\right)$.

F. $-\frac{3}{2}$

G. -2

H. $2\frac{1}{2}$

J. $-\frac{7}{2}$

3. Which point could not be a part of a function that includes $(0, -2)$, $(1, 1)$, $(-2, -8)$, $(-1, -5)$, and $(3, 7)$?

A. $(-3, -11)$

B. $(2, 4)$

C. $(4, 10)$

D. $(-1, 4)$

4. For the function $f(x) = 6x - 2$, which of the following is $f(-4)$?

F. -26

G. -24

H. 22

J. 26

5. What is the domain of the relation given by the ordered pairs $(0, -7)$, $(4, -3)$, $(6, -1)$, $(-1, -8)$, and $(5, -2)$?

A. $\{-8, -7, -3, -2, -1\}$

B. $\{-3, -2, -1, 4, 5\}$

C. $\{-7, -3, -1, 4, 5\}$

D. $\{-1, 0, 4, 5, 6\}$

6. What is the inverse of the function $f(x) = -2x - 4$?

F. $f^{-1}(x) = \frac{1}{2}x - 2$

G. $f^{-1}(x) = \frac{1}{2}x + 2$

H. $f^{-1}(x) = -\frac{1}{2}x - 2$

J. $f^{-1}(x) = 2x - 2$

7. What is the inverse of the relation below?

x	−2	−1	0	1
y	6	5	4	3

A.

x	0	3	4	6
y	4	1	0	−2

B.

x	3	4	5	6
y	1	0	−1	−2

C.

x	3	4	5	6
y	−2	−1	0	1

D.

x	2	3	4	5
y	−2	1	0	−3

8. Let $f(x) = 5x - 10$. Which of the following is $f^{-1}(x)$?

F. $f^{-1}(x) = \frac{1}{5}x + 10$

G. $f^{-1}(x) = \frac{1}{5}x + 5$

H. $f^{-1}(x) = \frac{1}{5}x + 2$

J. $f^{-1}(x) = x + 2$

9. What are the minimum and maximum values for the function $f(x) = \frac{4 - x}{3}$ on the interval [1, 13]?

A. minimum value: 1; maximum value: 3

B. minimum value: −1; maximum value: 3

C. minimum value: −3; maximum value: 1

D. minimum value: −3; maximum value: −1

10. What are the minimum and maximum values for the function $f(x) = 4x + 3$ on the interval [−2, 5]?

F. minimum value: −5; maximum value: 23

G. minimum value: 5; maximum value: 23

H. minimum value: −9; maximum value: 18

J. minimum value: −9; maximum value: 23

11. Which point could not be a part of a function that includes (0, 0), (−2, 4), (−1, 1), (3, 9), and (4, 16)?

A. (5, 25)

B. (−2, −4)

C. (−3, 9)

D. (−4, 16)

12. For the function $f(x) = \frac{1}{2}x - 2$, which of the following is $f(-6)$?

F. −5

G. −1

H. 1

J. 5

13. What is the y-intercept of the function $f(x) = 6 - 5x$?

A. (0, −5)

B. (0, −6)

C. $(0, 1\frac{1}{5})$

D. (0, 6)

14. What is the inverse of the relation below?

x	−4	−1	2	5
y	−2	1	4	7

F.

x	−2	1	4	7
y	−4	−1	2	5

G.

x	1	−2	1	7
y	−4	4	−1	5

H.

x	−2	−1	4	7
y	−1	1	5	2

J.

x	−1	1	4	5
y	−1	−1	2	7

Gridded Response

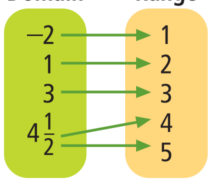
15. What is the minimum of $f(x) = 2x^2$?

16. What is $f(3)$ for the function $f(x) = -x - \frac{1}{2}$?

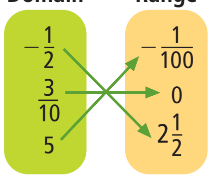
Constructed Response

Determine whether each relation is a function. Explain your reasoning.

17. Domain Range



18. Domain Range



19. What is $f(-3)$ for the function $f(x) = -3x + 6$?

20. Find $f(-3)$, $f(0)$, and $f(1)$ for the function $f(x) = \frac{2}{5}x - 2$.

21. The points $(-3, 2)$, $(-1, 3)$, $(0, 0)$, $(-2, -1)$ represent a function. What are the domain and range?

Let $f(x) = x - 2$ and $g(x) = x^2 - 3x + 2$. Perform each function operation and then find the domain.

22. $-2g(x) + f(x)$ **23.** $-f(x) \cdot g(x)$ **24.** $\frac{g(x)}{f(x)}$

For each pair of functions, find $(g \circ f)(x)$ and $(f \circ g)(x)$.

25. $f(x) = x^2 - 2$, $g(x) = 4x + 1$

26. $f(x) = 2x^2 + x - 7$, $g(x) = -3x - 1$

27. Let $g(x) = x - 3$ and $h(x) = x^2 + 6$. What is $h(1) \cdot g(1)$?

28. You are given that $f(x) = x^2 + 4$ and $g(x) = 3x - 1$.

a. What are the domain and range of $f(x)$ and $g(x)$?

b. Find $f(x) + g(x)$.

c. What is the domain of your answer to part (b)?

Find the domain and range. Graph each relation.

29. $\{(0, 0), (1, -1), (2, -4), (3, -9), (4, -16)\}$

30. $\{(3, 2), (4, 3), (5, 4), (6, 5), (7, 6)\}$

31. An online music store is having a promotion. Customers receive a \$5 rebate if they buy any regular priced CD at \$13 each. They can also receive 15% off if they register as a store member.

a. What functions model the two discounts?

b. In which order should the discounts be applied for the customer to receive the greatest discount?

c. Use your answer from part (b) to determine the amount a customer will save if she buys 5 CDs.

Topic 2 Absolute Value Equations and Functions

TOPIC OVERVIEW

2-1 Absolute Value Equations
2-2 Solving Absolute Value Inequalities
2-3 Attributes of Absolute Value Functions
2-4 Transformations of Absolute Value Functions
2-5 Graphing Absolute Value Inequalities

VOCABULARY

English/Spanish Vocabulary Audio Online:

English	Spanish
absolute value, *p. 36*	valor absoluto
absolute value inequality, *p. 58*	desigualdad de valor absolute
axis of symmetry, *p. 45*	eje de simetría
boundary, *p. 58*	límite
extraneous solution, *p. 36*	solución extraña
test point, *p. 58*	punto de prueba

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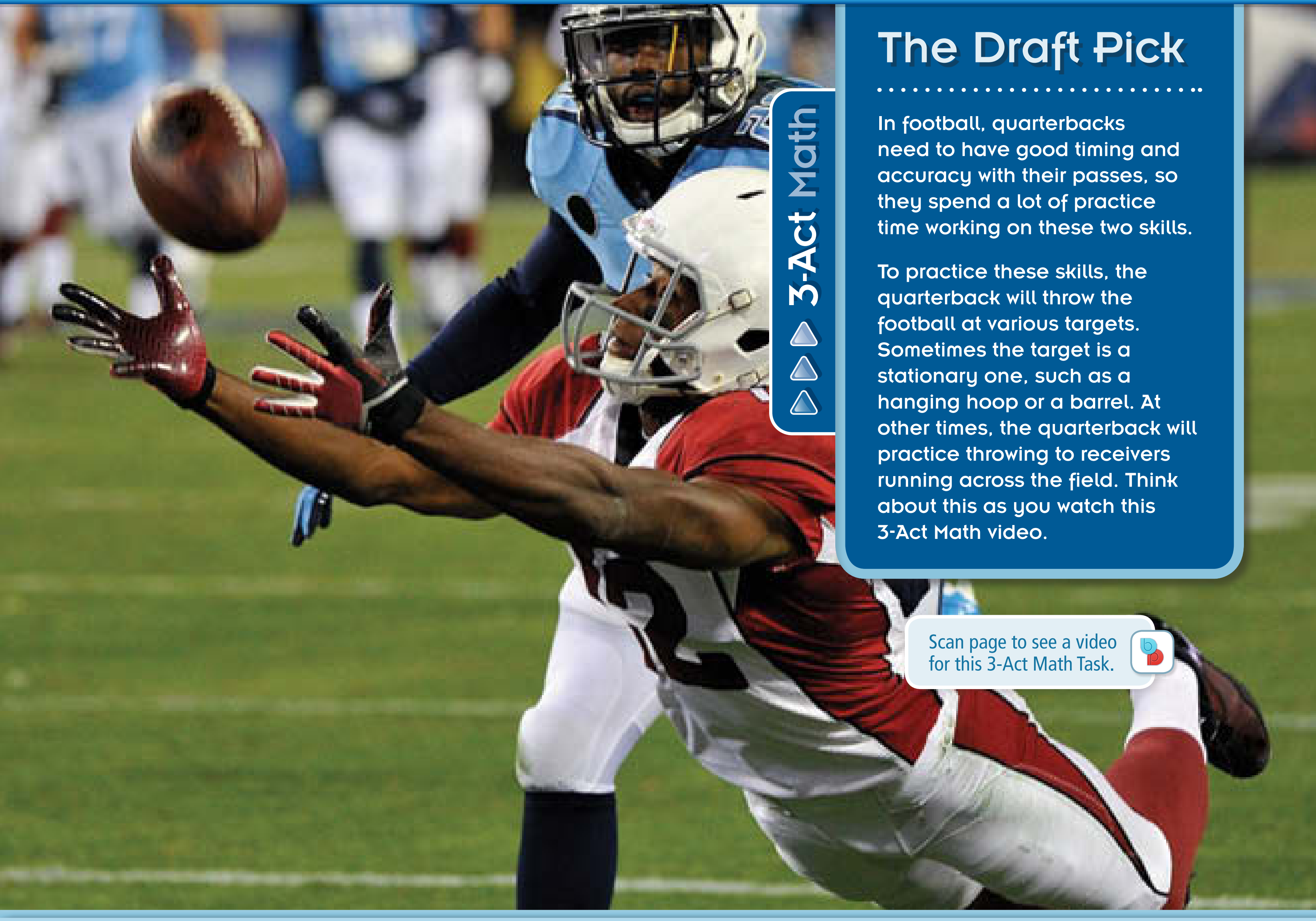

3-Act Math

The Draft Pick

In football, quarterbacks need to have good timing and accuracy with their passes, so they spend a lot of practice time working on these two skills.

To practice these skills, the quarterback will throw the football at various targets. Sometimes the target is a stationary one, such as a hanging hoop or a barrel. At other times, the quarterback will practice throwing to receivers running across the field. Think about this as you watch this 3-Act Math video.

Scan page to see a video for this 3-Act Math Task.

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2-1 Absolute Value Equations

TEKS FOCUS

TEKS (6)(E) Solve absolute value linear equations.

TEKS (1)(B) Use a problem–solving model that incorporates analyzing given information, **formulating** a plan or **strategy**, determining a solution, justifying the solution, and evaluating the problem–solving process and the **reasonableness** of the solution.

Additional TEKS (1)(D), (1)(E), (6)(D)

VOCABULARY

- **Absolute value** – The absolute value of a real number x, written $|x|$, is its distance from zero on a number line.
- **Extraneous solution** – An extraneous solution is a solution derived from an original equation that is *not* a solution to the original equation.
- **Formulate** – create with careful effort and purpose. You can formulate a plan or strategy to solve a problem.
- **Strategy** – a plan or method for solving a problem
- **Reasonableness** – the quality of being within the realm of common sense or sound reasoning. The reasonableness of a solution is whether or not the solution makes sense.

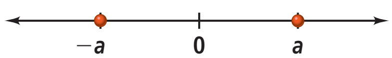
ESSENTIAL UNDERSTANDING

An absolute value quantity is nonnegative. Since opposites have the same absolute value, an absolute value equation can have two solutions.

take note

Key Concept Absolute Value

Definition	Numbers	Symbols
The **absolute value** of a real number x, written $\lvert x\rvert$, is its distance from zero on the number line.	$\lvert 4\rvert = 4$ $\lvert -4\rvert = 4$	$\lvert x\rvert = x$, if $x \geq 0$ $\lvert x\rvert = -x$, if $x < 0$

Absolute Value Equation	Meaning	Graph and Solution
$\lvert x\rvert = a$	the distance between x and 0 is a units	 $x = -a$ or $x = a$

Problem 1

TEKS Process Standard (1)(E)

Solving an Absolute Value Equation

Think

How is solving this equation different from solving a linear equation?
In the absolute value equation, $2x - 1$ can represent two opposite quantities.

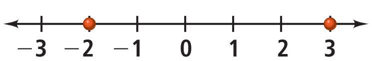
What is the solution of $|2x - 1| = 5$? Graph the solution.

$|2x - 1| = 5$

$2x - 1 = 5$ or $2x - 1 = -5$ Rewrite as two equations. $2x - 1$ could be 5 or -5.

$2x = 6$ $\quad$ $2x = -4$ Add 1 to each side of both equations.

$x = 3$ or $x = -2$ Divide each side of both equations by 2.



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Problem 1 *continued*

Check $|2(3) - 1| \stackrel{?}{=} 5$ $\qquad$ $|2(-2) - 1| \stackrel{?}{=} 5$

$|6 - 1| \stackrel{?}{=} 5$ $\qquad$ $|-4 - 1| \stackrel{?}{=} 5$

$|5| = 5$ ✔ $\qquad$ $|-5| = 5$ ✔

Problem 2

Plan

Is there a simpler way to think of this problem?
Solving $3|x + 2| - 1 = 8$ is similar to solving $3y - 1 = 8$.

Solving a Multi-Step Absolute Value Equation

What is the solution of $3|x + 2| - 1 = 8$? Graph the solution.

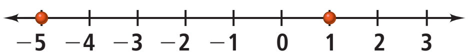
$3|x + 2| - 1 = 8$

$3|x + 2| = 9$ $\qquad$ Add 1 to each side.

$|x + 2| = 3$ $\qquad$ Divide each side by 3.

$x + 2 = 3$ or $x + 2 = -3$ $\qquad$ Rewrite as two equations.

$x = 1$ or $x = -5$ $\qquad$ Subtract 2 from each side of both equations.



Check $3|(1) + 2| - 1 \stackrel{?}{=} 8$ $\qquad$ $3|(-5) + 2| - 1 \stackrel{?}{=} 8$

$3|3| - 1 \stackrel{?}{=} 8$ $\qquad$ $3|-3| - 1 \stackrel{?}{=} 8$

$8 = 8$ ✔ $\qquad$ $8 = 8$ ✔

Problem 3

Think

Can you solve this the same way as you solved Problem 1?
Yes, let $3x + 2$ equal $4x + 5$ and $-(4x + 5)$.

Checking for Extraneous Solutions

What is the solution of $|3x + 2| = 4x + 5$? Check for extraneous solutions.

$|3x + 2| = 4x + 5$

$3x + 2 = 4x + 5$ or $3x + 2 = -(4x + 5)$ $\qquad$ Rewrite as two equations.

$-x = 3$ $\qquad$ $3x + 2 = -4x - 5$ $\qquad$ Solve each equation.

$7x = -7$

$x = -3$ or $x = -1$

Check $|3(-3) + 2| \stackrel{?}{=} 4(-3) + 5$ $\qquad$ $|3(-1) + 2| \stackrel{?}{=} 4(-1) + 5$

$|-9 + 2| \stackrel{?}{=} -12 + 5$ $\qquad$ $|-3 + 2| \stackrel{?}{=} -4 + 5$

$|-7| \neq -7$ ✘ $\qquad$ $|-1| = 1$ ✔

Since $x = -3$ does not satisfy the original equation, -3 is an extraneous solution. The only solution to the equation is $x = -1$.

Problem 4

TEKS Process Standard (1)(B)

Formulating an Absolute Value Equation

You drive 8 miles from home to a gym. After your workout, you drive to a farm that is *m* miles from your home on the same road as your home and the gym. After buying some fruits and vegetables, you drive back to the gym where you have left your cell phone. So far, you have driven a total of 15 miles. Formulate an absolute value equation to represent this situation.

Think	Write
You know the distance from home to the gym. The distance from home to the farm is *m*.	distance to gym = 8 distance to farm = m
The farm is *m* miles from home, in *either direction* from the gym. So, use absolute value to represent the distance from the gym to the farm.	distance from gym to farm = $\lvert 8 - m \rvert$
Write an equation. You traveled the distance between the gym and the farm twice.	total distance traveled $15 = 8 + \lvert 8 - m \rvert + \lvert 8 - m \rvert$

An absolute value equation representing this situation is $15 = 8 + 2\lvert 8 - m \rvert$.

PRACTICE and APPLICATION EXERCISES

ONLINE HOMEWORK

Scan page for a Virtual Nerd™ tutorial video.

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Solve each equation. Check your answers.

1. $\lvert 3x \rvert = 18$

2. $\lvert -4x \rvert = 32$

3. $\lvert x - 3 \rvert = 9$

4. $2\lvert 3x - 2 \rvert = 14$

5. $\lvert 3x + 4 \rvert = -3$

6. $\lvert 2x - 3 \rvert = -1$

7. $\lvert x + 4 \rvert + 3 = 17$

8. $\lvert 4 - z \rvert - 10 = 1$

Solve each equation. Check for extraneous solutions.

9. $\lvert x - 1 \rvert = 5x + 10$

10. $\lvert 2z - 3 \rvert = 4z - 1$

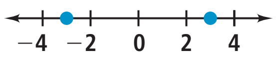
11. $\lvert 3x + 5 \rvert = 5x + 2$

12. $\lvert 2y - 4 \rvert = 12$

13. $3\lvert 4w - 1 \rvert - 5 = 10$

14. $\lvert 2x + 5 \rvert = 3x + 4$

15. Write an absolute value equation to describe the graph.



Solve each equation.

16. $-|4 - 8b| = 12$ **17.** $4\,|3x + 4| = 4x + 8$ **18.** $|3x - 1| + 10 = 25$

19. $\frac{1}{2}|3c + 5| = 6c + 4$ **20.** $5|6 - 5x| = 15x - 35$ **21.** $7|8 - 3h| = 21h - 49$

22. $6|2x + 5| = 6x + 24$ **23.** $\frac{1}{4}|4x + 7| = 8x + 16$ **24.** $\frac{2}{3}|3x - 6| = 4(x - 2)$

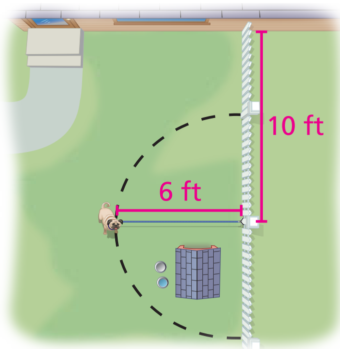
Create Representations to Communicate Mathematical Ideas (1)(E) **Write an absolute value equation that represents each set of numbers.**

25. all real numbers exactly 6 units from 0

26. all real numbers exactly 5 units from 3

Apply Mathematics (1)(A) **Write an absolute value equation to represent each situation.**

27. A fence extends from the side of a house. A dog is tied to a fence post ten feet from the side of the house. The dog's leash is six feet long. Let d be the greatest and least distance from the house that the dog can reach along the fence.



28. A factory produces widgets whose length must be within 1.5 mm of an ideal length of 47 mm. A factory supervisor wants to mark rulers with the least and greatest acceptable length for each widget for workers to use in quality control inspections. Let ℓ be the length of the markings along the ruler.

29. You and your friends Jose and Sarah all live on the same street. You know that Jose lives five blocks away from you. Jose says that Sarah lives two blocks from him, but he didn't say whether she lives closer to you than he does or further. Let b be the number of blocks Sarah might live from you.

Is the absolute value equation *always*, *sometimes*, or *never* true? Explain.

30. $|x| = -6$ **31.** $|x| = x$

32. $|x| + |x| = 2x$ **33.** $|x + 2| = x + 2$

Solve each equation for x.

34. $|ax| - b = c$ **35.** $|cx - d| = ab$ **36.** $a|bx - c| = d$

Write an absolute value equation that has the given solution.

37. −3 and 3 **38.** 2 and 4 **39.** −1 and 5

40. 3 and 4 **41.** 0 **42.** 7

43. −11 **44.** no solution **45.** infinitely many solutions

Use Multiple Representations to Communicate Mathematical Ideas (1)(D) **In the diagram, your office is shown as 12 miles from home. The question marks represent possible locations of a pizza parlor, which is x miles from your home. Use the diagram for Exercises 46–49.**

46. Formulate an absolute value equation to represent d, where d represents the distance from the office to the pizza parlor.

47. Formulate an absolute value equation to represent e, where e represents the total distance to drive from the office to the pizza parlor, and then back to the office.

48. If $f = 12 + |12 - x|$, describe in words what the distance f might represent.

49. Is it possible to formulate an absolute value equation to represent g, if g is the total distance driven from home to the office, to the pizza parlor, and then home? Explain.

TEXAS Test Practice

50. What is the positive solution of $|3x + 8| = 19$?

51. What is the solution of $|2x - 4| = 16$?

A. 10 **C.** 10, -6

B. 10, -10 **D.** 6, -6

52. How many solutions does the equation $|4x + 7| = -3$ have?

F. 0 **H.** 2

G. 1 **J.** infinitely many

53. The packing material for a particular computer needs to be within 0.5 mm of the desired thickness, which is 27.5 mm. Which equation represents the limits of the width of the packing material?

A. $|w + 27.5| = 0.5$ **C.** $|w + 0.5| = 27.5$

B. $|w - 0.5| = 27.5$ **D.** $|w - 27.5| = 0.5$

2-2 Solving Absolute Value Inequalities

TEKS FOCUS

TEKS (6)(F) Solve absolute value linear inequalities.

TEKS (1)(A) **Apply** mathematics to problems arising in everyday life, society, and the workplace.

Additional TEKS (1)(E), (1)(D)

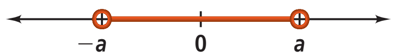
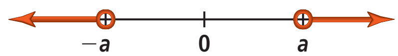
VOCABULARY

- **Apply** – use knowledge or information for a specific purpose, such as solving a problem

ESSENTIAL UNDERSTANDING

You can write an absolute value inequality as a compound inequality without absolute value symbols.

take note — Concept Summary: Solutions of Absolute Value Statements

Symbols	Definition	Graph
$\lvert x\rvert < a$ $(\lvert x\rvert \le a)$	The distance from x to 0 is less than a units.	 $-a < x < a$ $x > -a$ and $x < a$
$\lvert x\rvert > a$ $(\lvert x\rvert \ge a)$	The distance from x to 0 is greater than a units.	 $x < -a$ or $x > a$

Problem 1

TEKS Process Standard (1)(E)

Solving the Absolute Value Inequality $|A| < b$

What is the solution of $|2x - 1| < 5$? Graph the solution.

Plan

Is this an *and* problem or an *or* problem?
$2x - 1$ is less than 5 and greater than -5. It is an *and* problem.

$|2x - 1| < 5$

$-5 < 2x - 1 < 5$ — $2x - 1$ is between -5 and 5.

$-4 < 2x < 6$ — Add 1 to each part.

$-2 < x < 3$ — Divide each part by 2.

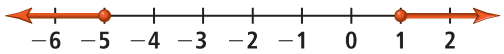
Problem 2

Solving the Absolute Value Inequality $|A| \geq b$

What is the solution of $|2x + 4| \geq 6$? Graph the solution.

$$|2x + 4| \geq 6$$

$2x + 4 \leq -6$	or	$2x + 4 \geq 6$	Rewrite as a compound inequality.
$2x \leq -10$		$2x \geq 2$	Subtract 4 from each side of both inequalities.
$x \leq -5$	or	$x \geq 1$	Divide each side of both inequalities by 2.



Think

How do you determine the boundary points?
To find the boundary points, find the solutions of the related equation.

Problem 3

TEKS Process Standard (1)(A)

Using an Absolute Value Inequality

Car Racing **In car racing, a car must meet specific dimensions to enter a race. Officials use a template to ensure these specifications are met. What absolute value inequality describes heights of the model of race car shown within the indicated tolerance?**

$\frac{53 - 51}{2} = \frac{2}{2} = 1$	Find the tolerance.
$-1 \leq h - 52 \leq 1$	Use h for the height of the race car. Write a compound inequality.
$\|h - 52\| \leq 1$	Rewrite as an absolute value inequality.

Plan

How does *tolerance* relate to an inequality?
Tolerance allows the height to differ from a desired height by no less and no more than a small amount.

PRACTICE and APPLICATION EXERCISES

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Solve each inequality. Graph the solution.

1. $3|y-9|<27$
2. $|6y-2|+4<22$
3. $|3x-6|+3<15$
4. $\frac{1}{4}|x-3|+2<1$
5. $4|2w+3|-7\leq 9$
6. $3|5t-1|+9\leq 23$

Solve each inequality. Graph the solution.

7. $|x+3|>9$
8. $|x-5|\geq 8$
9. $|y-3|\geq 12$
10. $|2x+1|\geq -9$
11. $3|2x-1|\geq 21$
12. $|3z|-4>8$

Write each compound inequality as an absolute value inequality.

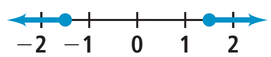
13. $1.3\leq h\leq 1.5$
14. $50\leq k\leq 51$
15. $27.25\leq C\leq 27.75$
16. $50\leq b\leq 55$
17. $1200\leq m\leq 1300$
18. $0.1187\leq d\leq 0.1190$

19. **Apply Mathematics (1)(A)** The circumference of a basketball for college women must be from 28.5 in. to 29.0 in. What absolute value inequality represents the circumference of the ball?

Write an absolute value inequality to describe each graph.

20.

21.



Solve each inequality. Graph the solutions.

22. $|3x-4|+5\leq 27$
23. $|2x+3|-6\geq 7$
24. $-2|x+4|<22$
25. $2|4t-1|+6>20$
26. $|3z+15|\geq 0$
27. $|-2x+1|>2$
28. $\frac{1}{9}|5x-3|-3\geq 2$
29. $\frac{1}{11}|2x-4|+10\leq 11$
30. $\left|\frac{x-3}{2}\right|+2<6$
31. $\left|\frac{x+5}{3}\right|-3>6$

32. **Analyze Mathematical Relationships (1)(F)** Describe the differences in the graphs of $|x|<a$ and $|x|>a$, where a is a positive real number.

33. **Create Representations to Communicate Mathematical Ideas (1)(E)** Write an absolute value inequality for which every real number is a solution. Write an absolute value inequality that has no solution.

Write an absolute value inequality to represent each situation.

34. **Apply Mathematics (1)(A)** Suppose you used an oven thermometer while baking and discovered that the oven temperature varied between $+5$ and -5 degrees from the setting. If your oven is set to 350°, let t be the actual temperature.

35. **Apply Mathematics (1)(A)** Workers at a hardware store take their morning break no earlier than 10 A.M. and no later than noon. Let c represent the time the workers take their break.

36. **Evaluate Reasonableness (1)(B)** A classmate wrote the solution to the inequality $|-4x + 1| > 3$ as shown. Describe and correct the error.

$$|-4x + 1| > 3$$
$$-4x + 1 > 3 \quad \text{or} \quad -4x + 1 < 3$$
$$-4x > 2 \quad \text{or} \quad -4x < 2$$
$$x < -\tfrac{1}{2} \quad \text{or} \quad x > -\tfrac{1}{2}$$

37. **Apply Mathematics (1)(A)** A friend is planning a trip to Alaska. He purchased a coat that is recommended for outdoor temperatures from $-15°F$ to $45°F$. Let t represent the temperature for which the coat is intended.

Write an absolute value inequality and a compound inequality for each length x with the given tolerance.

38. a length of 36.80 mm with a tolerance of 0.05 mm

39. a length of 9.55 mm with a tolerance of 0.02 mm

40. a length of 100 yd with a tolerance of 4 in.

Is the absolute value inequality *always, sometimes,* or *never* true? Explain.

41. $-8 > |x|$

42. $(|x|)^2 < x^2$

43. $|x| = x$

Graph each solution.

44. $|x| \geq 5$ and $|x| \leq 6$

45. $|x| \geq 6$ or $|x| < 5$

46. $|x - 5| \leq x$

47. **Connect Mathematical Ideas (1)(F)** Describe the difference between solving $|x + 3| > 4$ and $|x + 3| < 4$.

48. **Connect Mathematical Ideas (1)(F)** How can you determine whether an absolute value inequality is equivalent to a compound inequality joined by the word *and* or one joined by the word *or*?

TEXAS Test Practice

49. The normal thickness of a metal structure is shown. It expands to 6.54 centimeters when heated and shrinks to 6.46 centimeters when cooled down. What is the maximum amount in cm that the thickness of the structure can deviate from its normal thickness?

50. If p is an integer, what is the least possible value of p in the following inequality? $|3p - 5| \leq 7$

51. In wood shop, you have to drill a hole that is 2 inches deep into a wood panel. The tolerance for drilling a hole is described by the inequality $|t - 2| \leq 0.125$. What is the shallowest hole allowed?

2-3 Attributes of Absolute Value Functions

TEKS FOCUS

TEKS (2)(A) Graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple **representations**, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (1)(A), (7)(I)

VOCABULARY

- **Axis of symmetry** – The axis of symmetry is the line that divides a figure into two parts that are mirror images.
- **Parent Absolute Value Function** – The parent absolute value function is $f(x) = |x|$.
- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

Just as the absolute value of x is its distance from 0, the absolute value of $f(x)$, or $|f(x)|$, gives the distance from the line $y = 0$ for each value of $f(x)$. Graphing an absolute value function is one way to identify key attributes of the function, such as domain, range, intercepts, symmetries, and the maximum or minimum value of the function.

take note

Key Concept Absolute Value Parent Function $f(x) = |x|$

Table

x	$y = \|x\|$
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3

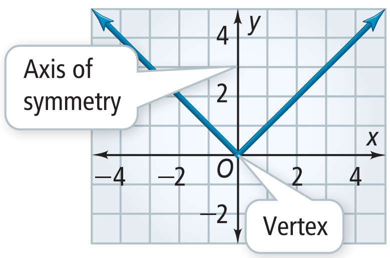
Function

$f(x) = |x|$

Note that $|x| = x$ when $x \geq 0$

and $|x| = -x$ when $x < 0$.

Graph



Domain	all real numbers
Range	$y \geq 0$
x-intercept	(0, 0)
y-intercept	(0, 0)

Problem 1 TEKS Process Standard (1)(D)

Domain, Range, and Intercepts of Absolute Value Functions

Graph the absolute value function and analyze the domain, range, and intercepts.

A $f(x) = |x|$

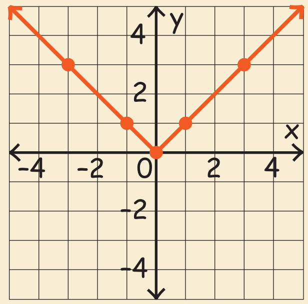
Think

Make a table of values and graph the function.

The domain of all absolute value functions is all real numbers. Use the location of the vertex to help you write the range.

Write

x	y
-3	3
-1	1
0	0
1	1
3	3



The domain is all real numbers.
The range is $y \geq 0$.
The x-intercept and the y-intercept are (0, 0).

B $f(x) = |x| + 2$

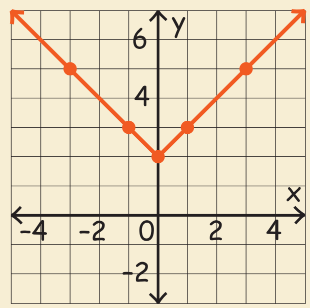
Think

Each *y*-value is 2 greater than the corresponding *y*-value in the table for Part A.

Since $|x|$ is always nonnegative, the value of $|x| + 2$ is always greater than or equal to 2. Since $|x| + 2$ is never equal to 0, there is no *x*-intercept.

Write

x	y
-3	5
-1	3
0	2
1	3
3	5



The domain is all real numbers.
The range is $y \geq 2$.
There is no x-intercept.
The y-intercept is (0, 2).

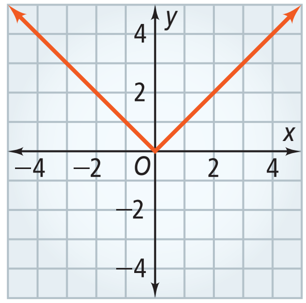
Problem 2

Symmetry in the Graphs of Absolute Value Functions

Graph the absolute value function and analyze the symmetry of the graph.

A $f(x) = |x|$

Use the graph from Problem 1 Part A.



The vertex is $(0, 0)$ and the axis of symmetry is a vertical line through the vertex.

The graph is symmetric about the axis of symmetry, which is the y-axis or $x = 0$.

Think

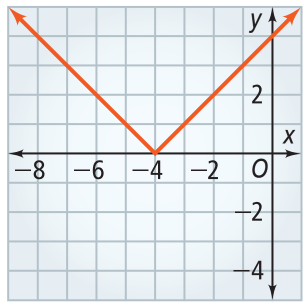
How can you write the equation of the axis of symmetry?
The equation of a vertical line always has the form $x = c$ for some real number c.

B $f(x) = |x + 4|$

Step 1 Make a table of values.

x	−7	−5	−4	−3	−1
$f(x)$	3	1	0	1	3

Step 2 Plot the points and draw the graph.



The vertex is $(-4, 0)$ and the axis of symmetry is a vertical line through the vertex.

The graph is symmetric about the axis of symmetry, which is the line $x = -4$.

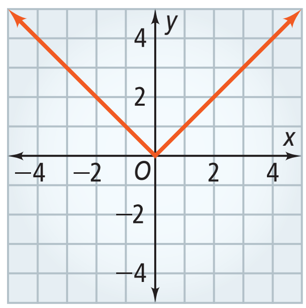
Problem 3

Finding the Maximum and Minimum of Absolute Value Functions

Graph the absolute value function and analyze the maximum and minimum on the given interval.

A $f(x) = |x|; [-4, 2]$

Use the graph from Problem 1 Part A.



The domain is $(-\infty, \infty)$. The range is $[0, \infty)$.

On the interval $[-4, 2]$, the minimum value of the function occurs at the vertex. Since the graph is V-shaped, the maximum occurs at one of the endpoints of the interval.

The minimum value on the interval is $f(0) = 0$ and the maximum value on the interval is $f(-4) = 4$.

Plan

How can you determine the maximum and minimum?
First determine the function's domain and range. Then restrict the domain to the given interval and find the least and greatest values of the function on the interval.

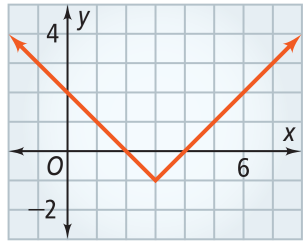
continued on next page ▶

Problem 3 *continued*

B $f(x) = |x - 3| - 1$; $[4, 7]$

Step 1 Make a table of values.

x	−1	1	3	5	7
$f(x)$	3	1	−1	1	3



Step 2 Plot the points and draw the graph.

The domain is $(-\infty, \infty)$. The range is $[-1, \infty)$.

On the interval $[4, 7]$, the function is an upward-sloping straight line, so the maximum and minimum values occur at the endpoints of the interval.

The minimum value on the interval is $f(4) = 0$ and the maximum value on the interval is $f(7) = 3$.

Problem 4

TEKS Process Standard (1)(A)

Interpreting Attributes of an Absolute Value Function

A ride at an amusement park carries passengers along a vertical tower to give them a view of their surroundings. During a test run, a park employee records the height of the ride for 10 s. The function $f(x) = |x - 5| + 4.8$ models the height $f(x)$ of the ride in meters after x seconds. Graph the function and interpret the domain, range, intercepts, symmetry, maximum, and minimum.

Step 1 Make a table of values.

x	0	2	5	8	10
$f(x)$	9.8	7.8	4.8	7.8	9.8

Think

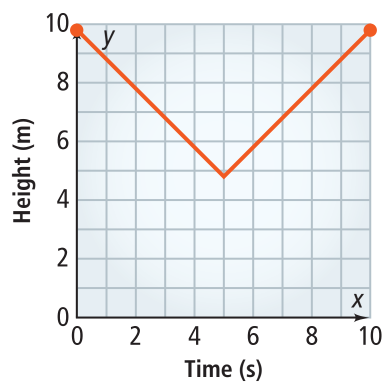
How can you determine the domain and range of a real-world function? Real-world functions usually have practical limitations to their domain and range. Read the problem carefully and think critically to find them.

Step 2 Plot the points and draw the graph.

Step 3 The domain is $[0, 10]$. The employee records the height for 10 s. The range is $[4.8, 9.8]$. The height of the ride ranges from 4.8 m to 9.8 m.

There is no x-intercept. The ride is never at ground level during this time period. The y-intercept is $(0, 9.8)$. When the employee starts recording heights, the ride is at a height of 9.8 m.

The axis of symmetry is $x = 5$. The ride is at the same height for any given number of seconds before and after 5 s.



The maximum height of the ride is 9.8 m. The minimum height of the ride is 4.8 m.

PRACTICE and APPLICATION EXERCISES

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Graph each absolute value function. Then analyze the domain, range, intercepts, and symmetry of the graph.

1. $f(x) = -|x|$ **2.** $f(x) = \frac{1}{2}|x| + 1$ **3.** $f(x) = |x - 2| - 2$ **4.** $f(x) = 2|x| - 4$

Graph each absolute value function. Then analyze the maximum and minimum on the given interval.

5. $f(x) = 2|x|; [-2, 1]$

6. $f(x) = |x + 1| - 3; [-4, -2]$

7. $f(x) = -|x| + 4; [0, 3]$

8. $f(x) = -|x - 2|; [-2, 3]$

Write the domain and range of each function in set notation and in interval notation.

9. $f(x) = |x| - 7.5$

10. $f(x) = |x - 1| + 4$

11. Use Multiple Representations to Communicate Mathematical Ideas (1)(D) You record the height of a kite for 9 seconds. The function $h(t) = |t - 2| + 12$ models the height h of the kite in meters after t seconds.

a. Use a table of values to graph the function. Then write the domain and range in interval notation and interpret the domain and range in this situation.

b. What are the maximum and minimum heights of the kite during the last 5 seconds that you record its height?

A student wrote the following statements in his notes about the function $f(x) = |x|$. Determine whether each statement is true or false. Explain.

12. The number -3 is in the domain of the function.

13. The number -3 is in the range of the function.

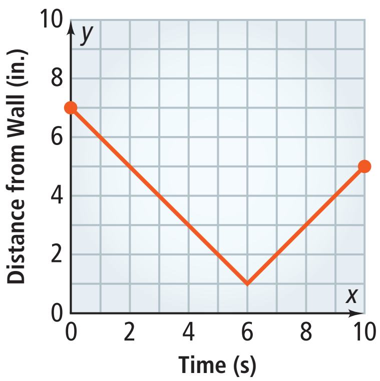
14. If you fold the graph of the function over the y-axis, the two halves of the graph will coincide.

15. The minimum value of the function on the interval $[-4, 3]$ is 3.

16. The maximum value of the function on the interval $[-3, 1]$ is 1.

17. The graph has the same x- and y-intercepts as the graph of $y = x$.

18. Apply Mathematics (1)(A) A remote-control car moves toward a wall, stops, backs up, and stops. The graph shows the car's distance from the wall as a function of time.



a. Write the domain and range in inequalities.

b. What are the x- and y-intercepts? What do these tell you about the motion of the car?

c. What are the maximum and minimum values of the function? What do these tell you about the motion of the car?

d. Write a function in the form $f(x) = |x - a| + b$ that models the motion of the car.

19. **Analyze Mathematical Relationships (1)(F)** Explain how the graph of the function $f(x) = |x|$ is similar to and different from the graph of the function $g(x) = x$. Be sure to discuss domain, range, intercepts, and symmetry.

20. **Explain Mathematical Ideas (1)(G)** A student said that an absolute value function of the form $f(x) = |x + a| + b$ has a y-intercept of $(0, b)$. Do you agree? Explain.

Determine whether each statement is always, sometimes, or never true.

21. An absolute value function of the form $f(x) = |x + a| + b$ has exactly one x-intercept.

22. An absolute value function of the form $f(x) = |x + a| + b$ has exactly one y-intercept.

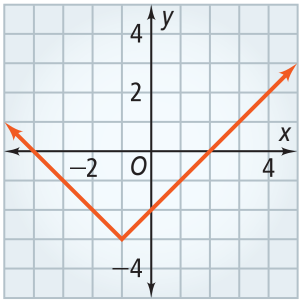
23. An absolute value function of the form $f(x) = c|x|$ is symmetric about the y-axis.

24. The range of the function $f(x) = c|x|$ is $[0, \infty)$.

25. If $b > 0$, then the graph of $f(x) = |x| + b$ intersects the x-axis.

26. The maximum value of the function $f(x) = |x|$ on the interval $[a, b]$ occurs at $x = b$.

27. The figure shows the graph of an absolute value function, $f(x)$.



   a. Write a function rule for $f(x)$.

   b. A student translates the graph of $f(x)$ up 3 units to produce the graph of $g(x)$. Write a function rule for $g(x)$.

   c. Explain how the domain and range of $f(x)$ compare to the domain and range of $g(x)$.

   d. Explain how the symmetry of the graph of $f(x)$ compares to the symmetry of the graph of $g(x)$.

TEXAS Test Practice

28. What is the minimum value of $f(x) = |x|$ on the interval $[-5, 3]$?

   A. -5 **B.** 0 **C.** 3 **D.** 5

29. Which function has the line $x = 12$ as its axis of symmetry?

   F. $f(x) = |x - 12|$ **G.** $f(x) = |x + 12|$ **H.** $f(x) = |x| + 12$ **J.** $f(x) = |x| - 12$

30. Which function has the same y-intercept as the function $f(x) = |x - 2| + 3$?

   A. $g(x) = |x + 1|$ **B.** $g(x) = |x| + 5$ **C.** $g(x) = |x| + 3$ **D.** $g(x) = |x + 3| - 2$

31. Which of the following is a true statement about the function $f(x) = -|x + 1| + 5$?

   F. The range of the function is $[5, \infty)$.

   G. The graph is symmetric about the line $x = 1$.

   H. The y-intercept is $(0, 5)$.

   J. There are two x-intercepts.

2-4 Transformations of Absolute Value Functions

TEKS FOCUS

TEKS (6)(C) Analyze the effect on the graphs of $f(x) = |x|$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(C), (2)(A)

VOCABULARY

- **Compression** – A compression is a transformation that decreases the distance between corresponding points of a graph and a line.
- **General form of the absolute value function** – a function of the form $f(x) = a|x - h| + k$
- **Reflection** – A reflection is a transformation that flips a graph across a line, such as the x- or y-axis.
- **Stretch** – A stretch is a transformation that increases the distance between corresponding points of a graph and a line.
- **Transformation** – A transformation of a function is a simple change to the equation of the function that results in a change in the graph of the function such as a translation or reflection.
- **Translation** – A translation is a transformation that shifts a graph vertically, horizontally, or both without changing its shape or orientation.

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

You can quickly graph absolute value functions by transforming the graph of $y = |x|$.

take note — Concept Summary: Absolute Value Function Family

Parent Function $f(x) = |x|$

Translation

$y = \|x\| + d$		$y = \|x - c\|$	
$d > 0$	shifts up $\|d\|$ units	$c > 0$	shifts to the right $\|c\|$ units
$d < 0$	shifts down $\|d\|$ units	$c < 0$	shifts to the left $\|c\|$ units

Stretch, Compression, and Reflection

$y = a\|x\|$		$y = \|bx\|$	
$\|a\| > 1$	vertical stretch	$\|b\| > 1$	horizontal compression (shrink)
$0 < \|a\| < 1$	vertical compression (shrink)	$0 < \|b\| < 1$	horizontal stretch
$a < 0$	reflection across x-axis	$b < 0$	reflection across y-axis

take note

Key Concept General Form of the Absolute Value Function

When a function has a vertex, the letters h and k are used to represent the coordinates of the vertex. Because an absolute value function has a vertex, the general form is $y = a|x - h| + k$. The vertical stretch or compression factor is $|a|$, the vertex is located at (h, k), and the axis of symmetry is the line $x = h$.

Problem 1

TEKS Process Standard (1)(E)

Analyzing the Graph of $f(x) + d$ When $f(x) = |x|$

What are the graphs of the absolute value functions $y = |x| - 4$ and $y = |x| + 1$? How are these graphs different from the parent function $f(x) = |x|$?

Make a table of values that you can use to compare the y-values of each transformed function to the y-values of the parent function.

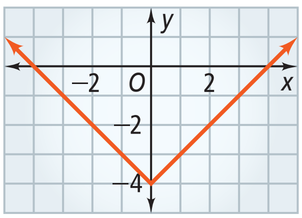
x	$\lvert x\rvert$	$\lvert x\rvert - 4$	$\lvert x\rvert + 1$
–3	3	–1	4
–1	1	–3	2
0	0	–4	1
1	1	–3	2
3	3	–1	4

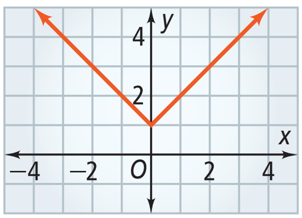
Think

How are you changing the y-coordinates to get the new graphs?
For $y = |x| - 4$, you are subtracting 4 from each y-coordinate of the graph of $y = |x|$, so the graph will move downward. For $y = |x| + 1$, you are adding 1, so the graph will move up.

To draw the graph of $y = |x| - 4$, you can move the entire graph of $y = |x|$ down 4 units without changing its shape. This is a shift or translation down 4 units. The vertex is now at $(0, -4)$ instead of $(0, 0)$.

For the graph of $y = |x| + 1$, you can translate the graph of $y = |x|$ up 1 unit. The vertex of this graph is at $(0, 1)$. The axis of symmetry is still the same in both graphs.





Problem 2

Analyzing the Graph of $f(x - c)$ When $f(x) = |x|$

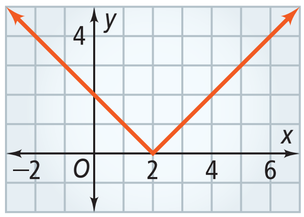
Plan

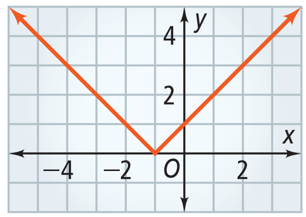
In Problem 1 you saw that the graph of $y = |x| + d$ translated the graph of $y = |x|$ up or down, depending on whether d was positive or negative. Make a table to see what adding or subtracting a number from x inside the absolute value does to the graph of the function.

What are the graphs of the absolute value functions $y = |x - 2|$ and $y = |x + 1|$? How are these graphs different from the parent function $f(x) = |x|$?

From the information in the table, you can see that the vertex of $y = |x - 2|$ is at the point $(2, 0)$. This means you can draw the graph of $y = |x - 2|$ by translating the graph of $y = |x|$ right 2 units. Similarly, if you translate the graph of $y = |x|$ left 1 unit, you produce the graph of $y = |x + 1|$.

x	$\lvert x\rvert$	$\lvert x - 2\rvert$	$\lvert x + 1\rvert$
–3	3	5	2
–2	2	4	1
–1	1	3	0
0	0	2	1
1	1	1	2
2	2	0	3
3	3	1	4
4	4	2	5





Problem 3

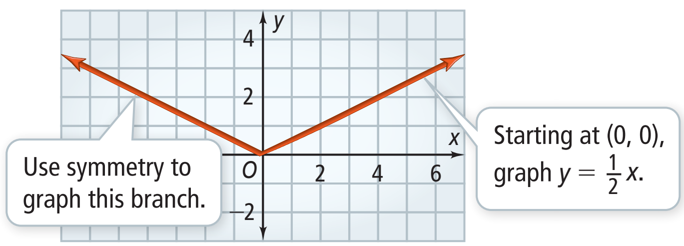
Analyzing the Graph of $af(x)$ When $f(x) = |x|$

Think

How can you describe the effect on the graph of $f(x) = |x|$? Use the equation to help you. The y-coordinate will be $\frac{1}{2}$ of what it was before.

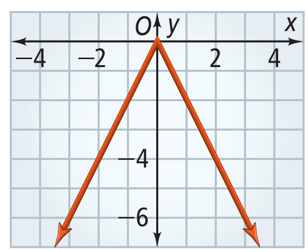
A **What is the graph of $y = \frac{1}{2}|x|$?**

The graph is a vertical compression of the graph of $f(x) = |x|$ by the factor $\frac{1}{2}$. Graph the right branch and use symmetry to graph the left branch.



B **What is the graph of $y = -2|x|$?**

Because the value of a is negative, the graph is reflected across the x-axis. Then the graph is a reflection of the graph of $f(x) = |x|$ followed by a vertical stretch by the factor 2.



Problem 4

TEKS Process Standard (1)(C)

Analyzing the Graph of $f(bx)$ When $f(x) = |x|$

Think

Which answer choices will help you consider graphs?
You can use paper and pencil or a graphing calculator to graph functions.

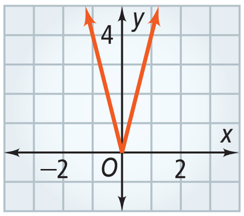
Consider how different values of b affect the graph of the function $f(bx)$ when $f(x) = |x|$. Use these values for b: 1, 4, $\frac{1}{4}$, and -4.

A Which tool would you use to analyze how changes in the value of b affect the graph of the function: real objects, manipulatives, paper and pencil, or technology? Why?

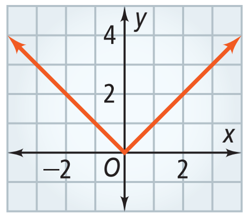
Use technology. With a graphing calculator you can graph all of the functions at once to quickly analyze how the different values of b affect the graph.

B Describe the effect on the graph by changing the value of b.

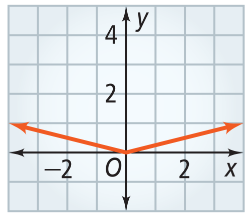
When $|b| > 1$, the graph of $f(x) = |x|$ is compressed horizontally to produce the graph of $f(x) = |bx|$.
When $0 < |b| < 1$, the graph of $f(x) = |x|$ is stretched horizontally to produce the graph of $f(x) = |bx|$.
The sign of b does not affect the graph, since the absolute value of any expression is always nonnegative.



$f(x) = |4x|$ or $f(x) = |-4x|$



$f(x) = |x|$



$f(x) = \left|\frac{1}{4}x\right|$

Problem 5

Identifying Transformations

Plan

To what should you compare $y = 3|x - 2| + 4$?
Compare it to the general form, $y = a|x - h| + k$.

Without graphing, what are the vertex, axis of symmetry, maximum or minimum, and x- and y-intercepts of the graph of $y = 3|x - 2| + 4$? How is the parent function $y = |x|$ transformed?

Compare $y = 3|x - 2| + 4$ with the general form $y = a|x - h| + k$.

$a = 3$, $h = 2$, and $k = 4$.

The vertex is (2, 4) and the axis of symmetry is $x = 2$. Because the value of a is positive, the y-coordinate of the vertex is the minimum point of the graph. The minimum is 4.

To find the x-intercept(s), set $y = 0$.

$$0 = 3|x - 2| + 4$$

$$-4 = 3|x - 2|$$

The equation $0 = 3|x - 2| + 4$ has no solutions, so there are no x-intercepts.

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Problem 5 *continued*

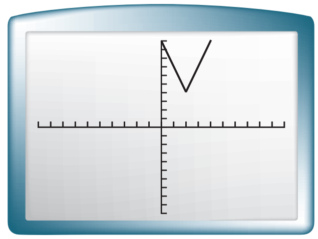
To find the y-intercept, set $x = 0$.

$$y = 3|0 - 2| + 4$$

$$y = 10$$

The y-intercept is (0, 10) or 10.

The parent function $y = |x|$ is translated 2 units to the right, vertically stretched by the factor 3, and translated 4 units up.

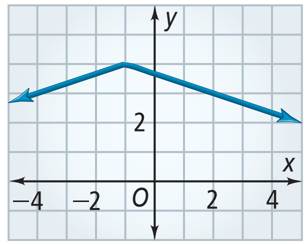


Check Check by graphing the equation on a graphing calculator.

Problem 6

Writing an Absolute Value Function

What is the equation of the absolute value function?



Think

What does the graph tell you about *a*?
The upside-down V suggests that $a < 0$.

Step 1 Identify the vertex.

The vertex is at $(-1, 4)$, so $h = -1$ and $k = 4$.

Step 2 Identify a.

The slope of the branch to the right of the vertex is $-\frac{1}{3}$, so $a = -\frac{1}{3}$.

Step 3 Write the equation.

Substitute the values of a, h, and k into the general form $y = a|x - h| + k$. The equation that describes the graph is $y = -\frac{1}{3}|x + 1| + 4$.

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Make a table of values for each equation. Then graph the equation. Analyze the effect on the graph of the parent function $f(x) = |x|$.

1. $y = |x| + 1$
2. $y = |x| - 1$
3. $y = |x| - \frac{3}{2}$
4. $y = |x + 2|$
5. $y = |x + 4|$
6. $y = |x - 2.5|$
7. $y = |x - 1| + 3$
8. $y = |x + 6| - 1$
9. $y = |x - 3.5| + 1.5$

Graph each equation. Then analyze the effect on the graph of the parent function $f(x) = |x|$.

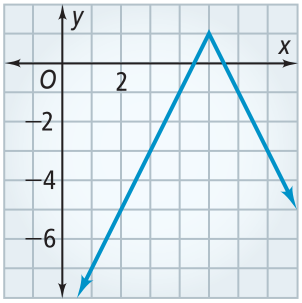
10. $y = 3|x|$
11. $y = -\frac{1}{2}|x|$
12. $y = -2|x|$
13. $y = \frac{1}{3}|x|$
14. $y = \frac{3}{2}|x|$
15. $y = -\frac{3}{4}|x|$

Without graphing, identify the vertex, axis of symmetry, and transformations from the parent function $f(x) = |x|$.

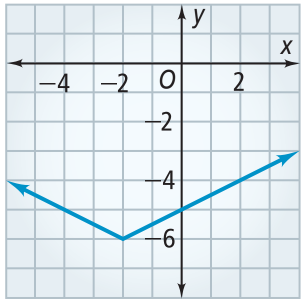
16. $y = |x + 2| - 4$
17. $y = \frac{3}{2}|x - 6|$
18. $y = 3|x + 6|$
19. $y = 4 - |x + 2|$
20. $y = -|x - 5|$
21. $y = |x - 2| - 6$

Write an absolute value equation for each graph.

22.



23.

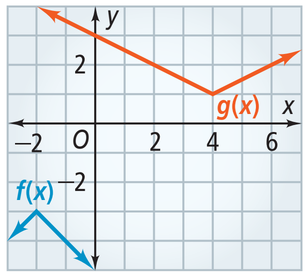


24. **Display Mathematical Ideas (1)(G)** Graph $y = -2|x + 3| + 4$. List the x- and y-intercepts, if any.

25. Graph $y = 4|x - 3| + 1$. List the vertex and the x- and y-intercepts, if any.

26. A classmate says that the graphs of $y = -3|x|$ and $y = |-3x|$ are identical. Graph each function and explain why your classmate is not correct.

27. The graphs of the absolute value functions $f(x)$ and $g(x)$ are given.



a. Describe a series of transformations that you can use to transform $f(x)$ into $g(x)$.

b. **Explain Mathematical Ideas (1)(G)** If you change the order of the transformations you found in part(a), could you still transform $f(x)$ into $g(x)$? Explain.

28. Graph each pair of equations on the same coordinate grid.

a. $y = 2|x + 1|$; $y = |2x + 1|$

b. $y = 5|x - 2|$; $y = |5x - 2|$

c. **Explain Mathematical Ideas (1)(G)** Explain why each pair of graphs in parts (a) and (b) are different.

Select Tools to Solve Problems (1)(C) **Use paper and pencil or a graphing calculator to compare the given graph to the parent function $f(x) = |x|$.**

29. $f(x) = |-x|$ **30.** $f(x) = \left|\frac{5}{2}x\right|$ **31.** $f(x) = |0.01x|$

32. Consider how different values of b affect the graph of the function $f(bx)$ when $f(x) = |x|$. What aspects of the parent function do not change for any value of b? What aspects change for particular values of b?

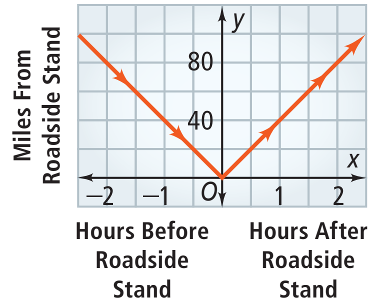
Graph each absolute value equation. Analyze the effect on the graph of the parent function $f(x) = |x|$.

33. $y = \left|-\frac{1}{4}x - 1\right|$ **34.** $y = \left|\frac{5}{2}x - 2\right|$ **35.** $y = \left|\frac{3}{2}x + 2\right|$

36. $y = |3x - 6| + 1$ **37.** $y = -|x - 3|$ **38.** $y = |2x + 6|$

39. $y = 2|x + 2| - 3$ **40.** $y = 6 - |3x|$ **41.** $y = 6 - |3x + 1|$

42. The graph at the right models the distance between a roadside stand and a car traveling at a constant speed. The x-axis represents time and the y-axis represents distance. Which equation best represents the relation shown in the graph?



A. $y = |60x|$ C. $y = |x| + 60$

B. $y = |40x|$ D. $y = |x| + 40$

43. a. Graph the equations $f(x) = -\frac{1}{2}|x - 3|$ and $g(x) = \left|-\frac{1}{2}(x - 3)\right|$ on the same set of axes.

b. **Analyze Mathematical Relationships (1)(F)** Describe the similarities and differences in the graphs.

44. a. Use a graphing calculator. Graph $y_1 = k|x|$ and $y_2 = |kx|$ for some positive value of k.

b. Graph $y_1 = k|x|$ and $y_2 = |kx|$ for some negative value of k.

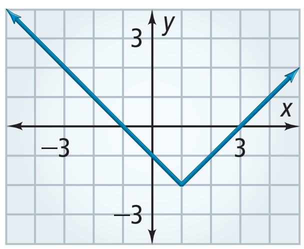
c. What conclusion can you make about the graphs of $y_1 = k|x|$ and $y_2 = |kx|$?

Graph each absolute value equation.

45. $y = |3x| - \frac{x}{3}$ 46. $y = \frac{1}{2}|x| + 4|x - 1|$ 47. $y = |2x| + |x - 4|$

TEXAS Test Practice

48. The graph at the right shows which equation?



A. $y = |3x - 1| + 2$ C. $y = |x - 1| - 2$

B. $y = |x - 1| + 2$ D. $y = |3x - 3| - 2$

49. How are the graphs of $y = 2x$ and $y = 2x + 2$ related?

F. The graph of $y = 2x + 2$ is the graph of $y = 2x$ translated down two units.

G. The graph of $y = 2x + 2$ is the graph of $y = 2x$ translated up two units.

H. The graph of $y = 2x + 2$ is the graph of $y = 2x$ translated to the left two units.

J. The graph of $y = 2x + 2$ is the graph of $y = 2x$ translated to the right two units.

50. What is the equation of a line parallel to $y = x$ that passes through the point (0, 1)?

A. $y = x + 1$ B. $y = 2x + 2$ C. $y = x - 1$ D. $y = -x$

51. Is $|y| = x$ a function? Explain.

2-5 Graphing Absolute Value Inequalities

TEKS FOCUS

TEKS (6)(C) Analyze the effect on the graphs of $f(x) = |x|$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (2)(A), (6)(F)

VOCABULARY

- **Absolute value inequality** – An absolute inequality in two variables has a graph that is a region of the coordinate plane with a V-shaped boundary.
- **Boundary** – The boundary of the graph of an absolute value inequality is a V-shape in the coordinate plane. It separates the solutions of the absolute value inequality from the non-solutions. Points on the boundary itself may or may not be solutions.
- **Test point** – To determine which region to shade, pick a test point that is not on the boundary. Check whether that point satisfies the absolute value inequality. If it does, shade the region that includes the test point. If not, shade the other region.
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

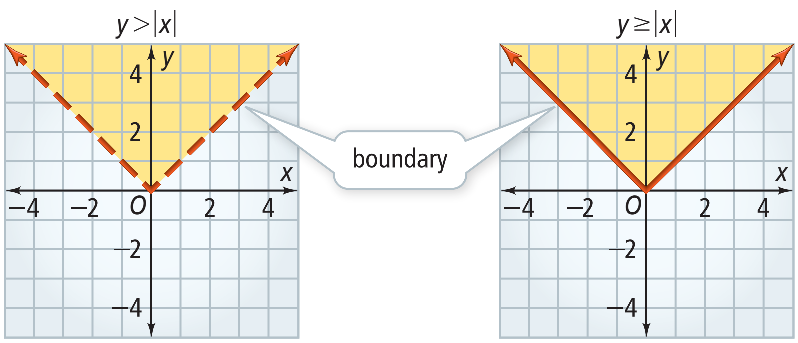
ESSENTIAL UNDERSTANDING

Graphing an absolute value inequality in two variables is similar to graphing a linear inequality. The graph of an absolute value inequality contains all points on one side of the V-shaped boundary and may or may not include the points on the boundary.

take note

Key Concept Absolute Value Inequality

An **absolute value inequality** in two variables has a graph that is a region of the coordinate plane with a V-shaped **boundary**.



To determine which region to shade, pick a **test point** that is not on the boundary. Check whether that point satisfies the inequality. If it does, shade the region that includes the test point. If not, shade the other region. The origin (0, 0) is usually an easy test point as long as it is not on the boundary.

Problem 1

TEKS Process Standard (1)(E)

Graphing an Absolute Value Inequality

What is the graph of $1 - y < |x + 2|$?

Know
Absolute value inequality

Need
Boundary

Plan
- Solve the inequality for y.
- Graph the related equation.
- Shade the solution.

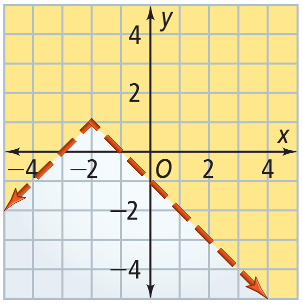
$$1 - y < |x + 2|$$

$$-y < |x + 2| - 1 \quad \text{Subtract 1 from each side.}$$

$$y > -|x + 2| + 1 \quad \text{Multiply both sides by } -1.$$

The graph of $y = -|x + 2| + 1$ is the graph of $y = |x|$, reflected in the x-axis and translated left 2 units and up 1 unit.

Since the inequality is solved for y and $y > -|x + 2| + 1$, shade the region above the boundary.

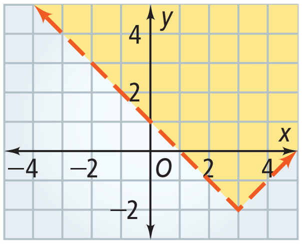


Problem 2

Writing an Inequality Based on a Graph

Plan
How can you tell that the graph is not a stretch or compression of the graph of $y = |x|$?
The slopes of the branches are 1 and −1.

What inequality does this graph represent?



The boundary is the graph of the absolute value function $y = |x|$, translated. The vertex of $y = |x|$ is translated to $(3, -2)$, so the boundary is the graph of $y = |x - 3| - 2$.

The solution is shaded above the boundary, so the inequality is either $>$ or $\geq$. Since the boundary is a dashed line, the correct inequality is $y > |x - 3| - 2$.

Problem 3

Solving an Absolute Value Inequality

What is the solution of the inequality $y > |x + 2| - 3$?

$y > |x+2| - 3$ Original inequality

$y + 3 > |x+2|$ Add 3 to both sides to isolate the absolute value.

Think

Does the definition of absolute value still work when there are two variables?
Yes. The definition of absolute value is for any real number, so it works for any algebraic expression that represents a real number.

An absolute value cannot be negative. Use the definition of absolute value to write one inequality when $x + 2 \geq 0$ and one when $x + 2 < 0$.

when $x + 2 \geq 0$, when $x + 2 < 0$,

$y + 3 > x + 2$ $\quad$ $y + 3 > -(x+2)$ Rewrite as a compound inequality.

$y > x - 1$ $\quad$ $y + 3 > -x - 2$ Solve each inequality for y.

$y > -x - 5$

Now combine the solutions to the two linear inequalities. $\begin{cases} y > -x - 5 & \text{for } x < -2 \\ y > x - 1 & \text{for } x \geq -2 \end{cases}$

Check The ordered pairs $(-6, 10)$ and $(4, 9)$ satisfy the algebraic solution. Do they satisfy the original inequality?

$10 > \|-6+2\| - 3$	$9 > \|4+2\| - 3$
$10 > \|-4\| - 3$	$9 > \|6\| - 3$
$10 > 4 - 3$	$9 > 6 - 3$
$10 > 1$ ✔	$9 > 3$ ✔

PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to **PearsonTEXAS.com**.

Graph each absolute value inequality.

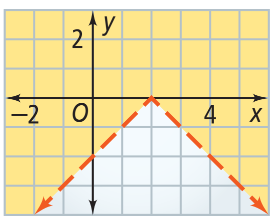
1. $y \geq |2x - 1|$ **2.** $y \leq |3x| + 1$ **3.** $y \leq |4 - x|$

4. $y > |-x + 4| + 1$ **5.** $y - 7 > |x + 2|$ **6.** $y + 2 \leq \left|\frac{1}{2}x\right|$

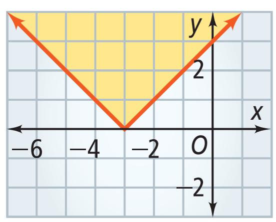
7. $3 - y \geq -|x - 4|$ **8.** $1 - y < |2x - 1|$ **9.** $y + 3 \leq |3x| - 1$

Write an inequality for each graph. The equation for the boundary line is given.

10. $y = -|x - 2|$



11. $2y = |2x + 6|$



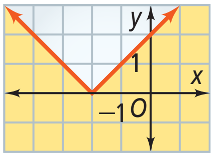
Graph each inequality on a coordinate plane.

12. $|x - 1| > y + 7$

13. $y - |2x| \le 21$

Write an inequality for each graph.

14.



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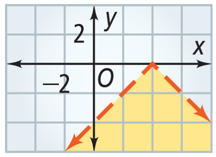

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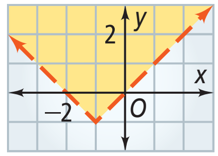
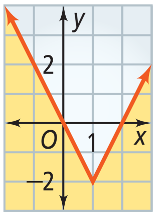


17. Select Techniques to Solve Problems (1)(C) When you graph an inequality, you can often use the point (0, 0) to test which side of the boundary line to shade. Describe a situation in which you could not use (0, 0) as a test point.

18. Which graph best represents the solution of the inequality $y \ge 2|x - 1| - 2$?

A.



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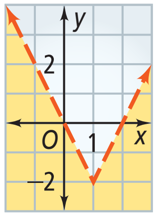
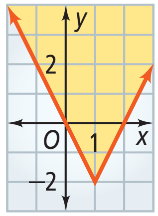
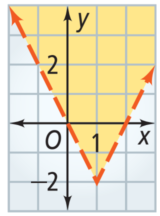


C.



D.



Select Tools to Solve Problems (1)(C) **Graph each inequality on a graphing calculator. Then sketch the graph.**

19. $y \le |x + 1| - |x - 1|$

20. $y > |x| + |x + 3|$

21. $y < |x - 3| - |x + 3|$

22. $y < 7 - |x - 4| + |x|$

Solve each absolute value inequality algebraically.

23. $y > |x - 3| + 5$

24. $y > |2x| - 8$

25. $y \le |x - 1| - 6$

26. $y + 7 < |x + 6|$

TEXAS Test Practice

27. Suppose y varies directly with x. If x is 30 when y is 10, what is x when y is 9?

A. 3 **B.** 27 **C.** 29 **D.** $\frac{300}{9}$

28. Which equation represents a line with slope -2 and y-intercept 3?

F. $3y = x - 2$ **G.** $3y = -2x + 1$ **H.** $y = 2x - 3$ **J.** $y = -2x + 3$

29. What is the vertex of $y = |x| - 5$?

A. (5, 0) **B.** (−5, 0) **C.** (0, 5) **D.** (0, −5)

30. The amount of a commission is directly proportional to the amount of a sale. A realtor received a commission of \$48,000 on the sale of an \$800,000 house. How much would the commission be on a \$650,000 house?

Topic 2 Review

TOPIC VOCABULARY

- absolute value, *p. 36*
- parent absolute value function, *p. 45*
- absolute value inequality, *p. 58*
- axis of symmetry, *p. 45*
- boundary, *p. 58*
- extraneous solution, *p. 36*
- test point, *p. 58*
- transformation, *p. 51*

Check Your Understanding

Choose the correct term to complete each sentence.

1. A number's distance from zero on the number line is its __?__.

2. A solution derived from the original equation but is not a solution to the original equation is a(n) __?__.

3. A(n) __?__ divides a graph into two parts that are mirror images.

4. A(n) __?__ of a function is a simple change to the equation of the function that results in a change in the graph of the function such as a translation or reflection.

2-1 and 2-2 Absolute Value Equations and Inequalities

Quick Review

To rewrite an equation or inequality that involves the **absolute value** of an algebraic expression, you must consider both cases of the definition of absolute value.

Example

Solve $|3x - 5| = 4 + 2x$. Check for extraneous solutions.

$3x - 5 = 4 + 2x$ or $3x - 5 = -(4 + 2x)$

$3x - 5 = -4 - 2x$

$5x = 1$

$x = 9$ or $x = \frac{1}{5}$

Check $|3(9) - 5| \stackrel{?}{=} 4 + 2(9)$ $\quad$ $\left|3\left(\frac{1}{5}\right) - 5\right| \stackrel{?}{=} 4 + 2\left(\frac{1}{5}\right)$

$|27 - 5| \stackrel{?}{=} 22$ $\quad$ $\left|\frac{3}{5} - 5\right| \stackrel{?}{=} 4 + \frac{2}{5}$

$|22| = 22$ ✔ $\quad$ $\left|-\frac{22}{5}\right| = \frac{22}{5}$ ✔

Exercises

Solve each equation. Check for extraneous solutions.

5. $|2x + 8| = 3x + 7$

6. $|x - 4| + 3 = 1$

7. $3|x + 10| = 6$

8. $2|x - 7| = x - 8$

Solve each inequality. Graph the solution.

9. $|3x - 2| + 4 \leq 7$

10. $4|y - 9| > 36$

11. $|7x| + 3 \leq 21$

12. $\frac{1}{2}|x + 2| > 6$

13. The specification for a length x is 43.6 cm with a tolerance of 0.1 cm. Write the specification as an absolute value inequality.

2-3 Attributes of Absolute Value Functions

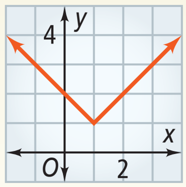
Quick Review

For any absolute value function, the domain is the set of all real numbers. The range has either a lower bound and the function has a minimum, or an upper bound and the function has a maximum.

The vertex is the point where the function reaches its maximum or minimum value. The **axis of symmetry** is a vertical line through the vertex.

Example

Graph the absolute value function $f(x) = |x - 1| + 1$ and analyze the maximum and minimum on the interval $[-1, 2]$.



On the interval $[-1, 2]$, the minimum occurs at the vertex, so the minimum value of the function is $f(1) = 1$.

Since the graph is V-shaped, the maximum occurs at one of the endpoints of the interval, $f(-1) = 3$.

Exercises

Graph each absolute value function. Then analyze the domain, range, intercepts, and symmetry of the graph.

14. $f(x) = |-x|$

15. $f(x) = |x + 2|$

16. $f(x) = -|x| + 2$

17. $f(x) = |x - 1| - 3$

Find the minimum and maximum values of $f(x)$ on the given interval.

18. $f(x) = |x|; [-15, 1]$

19. $f(x) = |x - 4| + 4; [0, 3]$

20. $f(x) = \frac{1}{2}|x| + 3; [-2, 4]$

21. $f(x) = |x| - 7; [-8, 0]$

22. Give an example of an absolute value function that has the line $x = 5$ as its axis of symmetry.

23. A student states that the graph of every absolute value function passes through the first quadrant. Do you agree? Justify your answer.

2-4 Transformations of Absolute Value Functions

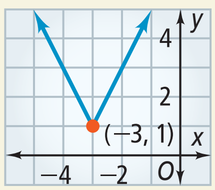
Quick Review

The function $y = |x|$ is the **parent absolute value function** for the family of functions of the form $y = a|x - h| + k$. The maximum or minimum point of the graph is the vertex of the graph.

$y = 2|x + 3| + 1$

$a = 2, h = -3, k = 1$

- Vertex is at $(-3, 1)$
- Translated left 3 units
- Stretched by a factor of 2
- Translated up 1 unit



Example

Write an equation for the translation of the graph $y = |x|$ up 5 units.

Because the graph is translated up, k is positive, so the equation of the translated graph is $y = |x| + 5$.

Exercises

Write an equation for each translation of the graph of $y = |x|$.

24. up 4 units, right 2 units

25. vertex $(-3, 0)$

26. vertex $(5, 2)$

27. vertex $(4, 1)$

Graph each function.

28. $f(x) = |x| - 8$

29. $f(x) = 2|x - 5|$

30. $y = -\frac{1}{4}|x - 2| + 3$

31. $y = -2|x + 1| - 1$

Without graphing, identify the vertex and axis of symmetry of each function.

32. $y = 2|x - 4|$

33. $y = -|x| + 2$

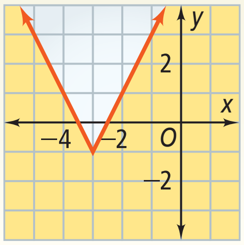
2-5 Graphing Absolute Value Inequalities

Quick Review

An inequality describes a region of the coordinate plane that has a **boundary**. To graph an inequality involving two variables, first graph the boundary. Then determine which side of the boundary contains the solutions. Points on a dashed boundary are not solutions. Points on a solid boundary are solutions.

Example

Graph the inequality $y \leq 2|x + 3| - 1$.



Graph the solid boundary $y = 2|x + 3| - 1$.

Since y is *less than* $2|x + 3| - 1$, shade below the boundary.

Exercises

Graph each inequality.

34. $y < -|x - 5|$

35. $y > |2x + 1|$

36. Write an absolute value inequality with a solid boundary that only has solutions below the x-axis.

Graph each inequality on a coordinate plane.

37. $y \geq |x + 4| - 6$

38. $y \leq 2|x| + 7$

39. $|x - 2| \leq y + 5$

40. $y + |3x| > 6$

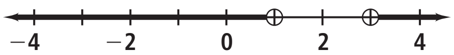
41. Solve the inequality $y \geq |x - 3| + 4$ algebraically.

Topic 2 TEKS Cumulative Practice

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. Which inequality has a solution that matches the graph below?



A. $|x-2|-3>-2$

B. $|x-2|<-5$

C. $|x+2|+3<2$

D. $|x+4|+2>1$

2. You used an oven thermometer while baking and found out that the oven temperature varied between $+7$ degrees and -7 degrees from the setting. If your oven is set to 325°F, let t be the actual temperature. What is the absolute value inequality that represents this situation?

F. $|t-325|\geq 7$

G. $|t-325|<7$

H. $|t-7|\leq 325$

J. $|t-325|\leq 7$

3. A designer is designing a handbag. The height of the handbag must be between 16 in. and 18 in. The desirable height is 17 in. Which absolute value inequality represents the height of the handbag?

A. $|h-16|\leq 1$

B. $|h-17|\leq 1$

C. $|h-17|\geq 2$

D. $|h-18|\geq 2$

4. How many negative solutions does $2|3x-6|\leq 6$ have?

F. 0

G. 1

H. 2

J. infinitely many

5. For which value of a does $4=a+|x-4|$ have no solution?

A. -6

B. 0

C. 4

D. 6

6. Which of the following absolute value inequalities has no solutions in Quadrant IV?

F. $y+2\geq|x-3|$

G. $y>3-|5-x|$

H. $y-1>|2x+6|$

J. $y\leq|4x|-7$

7. For which value of b would the equation $3|x-2|=bx-6$ have infinitely many solutions?

A. -6

B. 3

C. -3

D. 6

8. A meteorologist predicts the daily high and low temperatures as 91°F and 69°F. If t represents the temperature, then this situation can be described with the inequality $69\leq t\leq 91$. Which of the following absolute value inequalities is an equivalent way of expressing this?

F. $69\leq|t|\leq 91$

G. $|t-80|\leq 11$

H. $|t-69|\leq 91$

J. $|t-11|\leq 80$

9. Which inequality has a solution that matches the graph below?

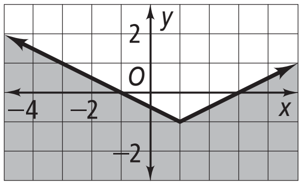
A. $|x-2|-3\geq 2$

B. $|x-2|-3\leq 2$

C. $|x-3|+2\geq 7$

D. $|x-3|+2\leq 7$

10. Which inequality best describes the graph?



F. $y \leq \frac{1}{2}|x+1| - 1$

G. $y \geq \frac{1}{2}|x-1| - 1$

H. $y \leq \frac{1}{2}|x-1| - 1$

J. $y \geq \frac{1}{2}|x+1| - 1$

11. Which describes the translation of $y = |x-3| + 5$?

A. $y = |x|$ translated 3 units left and 5 units up

B. $y = |x|$ translated 3 units right and 5 units up

C. $y = |x|$ translated 5 units left and 3 units up

D. $y = |x|$ translated 5 units right and 3 units up

12. Which of the following ordered pairs is a solution to the inequality $y > \frac{1}{3}|x-4| + 2$?

F. $(5, -1)$ **H.** $(-1, 3)$

G. $(1, 3)$ **J.** $(2, 4)$

13. Which of the following is *not* an important attribute of absolute value functions?

A. Vertex

B. Axis of symmetry

C. Rate of change

D. y-intercept

14. The graph of which equation is the graph of $f(x) = |x|$ reflected in the x-axis, translated 8 units left, vertically compressed by a factor of $\frac{1}{2}$, and translated down 5 units?

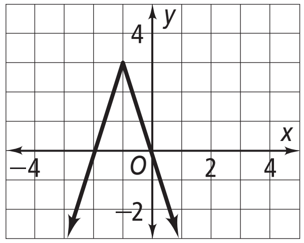
F. $y = -2|x-8| - 5$

G. $y = -\frac{1}{2}|x-8| - 5$

H. $y = 2|x+8| - 5$

J. $y = -\frac{1}{2}|x+8| - 5$

15. Which equation is graphed?



A. $y = -3|x+1| + 3$

B. $y = 3|x+1| + 3$

C. $y = -3|x-1| + 3$

D. $y = 3|x+1| - 3$

16. How many x-intercepts does the graph of $y = 4|x-12| - 9$ have?

F. 0 **H.** 2

G. 1 **J.** 3

17. What is the y-intercept of $y = 5|x-2| - 7$?

A. 2 **C.** 7

B. 3 **D.** −2

18. What is the axis of symmetry of $y = \frac{1}{4}|x+6| - 3$?

F. $x = -3$

G. $y = -3$

H. $x = 6$

J. $x = -6$

19. What is the equation of an absolute value function with vertical stretch factor $\frac{1}{3}$ and vertex $(-3, 2)$?

A. $y = \frac{1}{3}|x+3| + 2$

B. $y = \frac{1}{3}|x-3| + 2$

C. $y = 3|x-3| + 2$

D. $y = \frac{1}{3}|x+3| - 2$

Gridded Response

20. What is the sum of the solutions of $|2x+4|-6=8$?

21. What is the greatest integer solution of $|-2x-5|-3\leq 2$?

22. What is the sum of the solutions of $3|y-4|=9$ and $|y-4|=3$?

23. What is the minimum of $f(x)=2|x+1|-3$?

24. What is the horizontal translation of $f(x)=3|x-5|+7$?

25. What is the vertical stretch or compression factor of $f(x)=\frac{1}{2}|x+2|-1$?

26. What is the x-coordinate of the vertex of $f(x)=|x+9|+5$?

27. If the point $(a, 6)$ is an integer solution to $y< -|x-3|+7$, what is the value of a?

Constructed Response

28. A new 10-lb dumbbell will pass inspection if it is between 9.95 lb and 10.05 lb. What is the tolerance of the weight of the dumbbell? What absolute value inequality describes acceptable weights of the dumbbell within an indicated tolerance? Show all work.

Solve each equation or inequality. Graph the solution.

29. $4-x=|2-3x|$

30. $5|3w+2|-3>7$

Describe each transformation of the parent function $y=|x|$. Then, graph each function.

31. $y=|x|-4$

32. $y=|x-1|-5$

33. $y=-|x+4|+3$

34. $y=2|x+1|$

Graph each inequality.

35. $y\geq x+7$

36. $y>2|x+3|-3$

37. $4x-3y<2$

38. $y\leq -\frac{1}{2}|x+2|-3$

39. Graph $y<|x+3|$. Identify the parent function of the boundary and describe the translation.

40. Solve the inequality $y\leq 2|x-3|-4$ algebraically.

41. Graphs of absolute value functions consist of portions of two lines. What are the equations of the two lines that appear in the graph of $y=2|x+1|-1$? For what values of x does each appear?

42. **a.** Write an absolute value equation that has exactly two solutions.

b. Write an absolute value equation that has exactly one solution.

c. Write an absolute value equation that has no solutions.

d. How can you tell when an absolute value equation of the form $|x|=a$ will have exactly two solutions? Exactly one solution? No solutions?

43. The absolute value of a real number is always nonnegative. Is the output of an absolute value function always nonnegative? Why or why not? Justify your answer with an example.

44. Describe the transformations of $y=-7|x+41|-13$ from the parent function $f(x)=|x|$.

Additional Practice

Topic 1

Lesson 1-1

Determine whether each relation is a function. Justify your answer.

1. {(1, 4), (3, 2), (5, 2), (1, −8), (6, 7)}

2. {(7, 4), (4, 9), (−3, 1), (1, 7), (2, 8)}

Evaluate each function for the given value of *x*, and write the input *x* and output *f*(*x*) in an ordered pair.

3. $f(x) = -2x + 11$ for $x = 5$

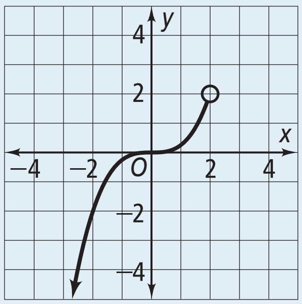
4. $f(x) = \frac{4}{3}x - \frac{2}{3}$ for $x = 8$

State the domain and range of each function using the specified notation.

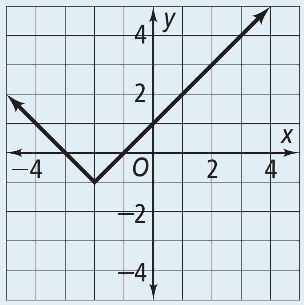
5. $f(x) = -x + 5$, interval notation

6. $f(x) = 2x - 3$, set notation

7. interval notation



8. set notation



9. Make a mapping diagram and a graph for the relation {(1, 3), (−2, −2), (1, 4), (0, 1)}. Then determine whether the relation is a function.

10. A garden has the shape of an isosceles triangle. The base of the triangle is 42 ft long, with an altitude of *h* ft. Write an equation that describes the area *A* of the garden as a function of *h*. State the domain and range of the function using interval notation. What is the area of the garden for $h = 38$ ft?

11. To avoid air turbulence, a plane climbs from an altitude of 20,700 feet to a higher altitude at the rate of 325 feet per minute. Write a function that describes the altitude, *y*, of the plane *x* minutes after it begins its climb. State the domain and range of the function using set notation. Then find the altitude of the plane for $x = 5$.

Lesson 1-2

Graph each function over the interval [−5, 5]. Find the domain, range, and intercepts of the function, then find the maximum and minimum values of *f*(*x*) over this interval.

12. $f(x) = x - 1$

13. $f(x) = \frac{4 + x}{2}$

14. $f(x) = \frac{1}{25}x^3$

Graph each function. Analyze the graph to determine the domain, range, asymptotic behavior, intercepts, and symmetry.

15. $f(x) = \frac{1}{x} + 2$

16. $f(x) = \frac{4}{x}$

17. $f(x) = 1 - \frac{3}{x}$

Lesson 1-2 *continued*

18. What are the horizontal and vertical asymptotes of the function $f(x) = \frac{1}{x-4} - 1$?

Lesson 1-3

Let $f(x) = 3x^2$ and $g(x) = 2 - 5x$. Perform each function operation. State the domain of the resulting function.

19. $f(x) - g(x)$ **20.** $f(x) \cdot g(x)$ **21.** $\frac{f(x)}{g(x)}$

22. $(f + g)(x)$ **23.** $(f \cdot g)(x)$ **24.** $\frac{g}{f}(x)$

Let $f(x) = x^2$ and $g(x) = 3x + 1$. Evaluate each expression.

25. $(f \circ g)(0)$ **26.** $(f \circ g)(2)$ **27.** $(g \circ f)(0)$

28. $(g \circ f)(-1)$ **29.** $(f \circ f)(3)$ **30.** $(g \circ g)(4)$

31. Halina works in a department store. Three times per year she is allowed to combine her employee discount with special sale prices. Let x be the retail price of a blouse.

a. Halina's employee discount is 20%. Write a function $E(x)$ that represents the cost of the blouse after the discount.

b. Due to a manufacturer's incentive, the blouse is marked down 25%. Write a function $M(x)$ that represents the sale price.

c. The sales tax on clothing is 6%. Write a function $T(x)$ that describes the cost of a clothing item with sales tax included.

d. Halina found a blouse to which the discounts apply. Use the function composition $f = T \circ E \circ M$ to write the function $f(x)$ that represents the price Halina will pay for the blouse.

32. You invest p dollars in an account that earns a simple interest of 6%. The function that represents the account balance at the end of the year is $f(p) = 1.06p$.

a. Suppose that at the end of the year you deposit \$500 in the account. Write a new function $g(p)$ that shows the balance that will earn interest in the second year.

b. At the end of every year you add \$500 to the account. The interest rate remains 6%. Write a composition of functions f and g to find the account balance at the end of the third year, before adding the \$500. Find that balance for an initial investment of \$1000.

Lesson 1-4

Find the inverse of the given functions.

33. $\{(2, 1), (3, 5), (0, -2), (-1, -1), (-4, 6)\}$ **34.** $f(x) = \frac{1}{2}x + 1$

35. $f(x) = 5x + 2$ **36.** $p(x) = \frac{5 - x}{3}$

Graph each function and its inverse on the same coordinate grid.

37. $f(x) = -x + 1$ **38.** $f(x) = x^2 + 2$ **39.** $f(x) = \pm\sqrt{x} - 1$

Topic 2

Lesson 2-1

Solve each equation. Check your answers.

1. $|4m + 2| = 10$
2. $|9 - 4z| = 53$
3. $|5x| = 30$
4. $|3x - 6| - 7 = 14$
5. $3|2d - 1| = 21$
6. $|2v + 3| - 6 = 14$
7. $4|3w - 1| = 2w$
8. $3|q + 6| = 4q - 1$
9. $y + 1 = 2|-4y + 2|$

Lesson 2-2

Solve each inequality. Graph the solution.

10. $|3 - k| < 7$
11. $|2t + 7| \geq 4$
12. $|x - 2| < 6$
13. $2|w + 6| \leq 10$
14. $|3y - 5| + 6 > 15$
15. $3|2z + 5| + 2 \leq 8$
16. $\left|\frac{-q + 4}{5}\right| + 1 > 2$
17. $\frac{1}{4}|x + 2| \leq 1$
18. $4\left|\frac{5z - 8}{2}\right| + 1 < 9$

19. A metal part for a machine is now 5.85 inches long. The specifications call for it to be 5.72 inches long, with a tolerance of ± 0.02 inch. By how much can a machinist decrease the length of the part?

Lesson 2-3

Graph each absolute value function over the interval $[-5, 5]$. Find the domain, range, and intercepts of the function, find the maximum and minimum values of $f(x)$ over this interval, and analyze the symmetry of the graph.

20. $f(x) = |x| - 1$
21. $f(x) = -\frac{1}{5}|x|$
22. $f(x) = 3 - |x|$
23. $f(x) = |x| + 0.5$
24. $f(x) = |x - 1| - 3$
25. $f(x) = 0.5|x + 2| - 1$

26. A hiker starts out at an elevation of 400 feet above sea level, and hikes for three hours. The function $e(t) = |300 - 150t| + 100$ models the elevation of the hiker after t hours of hiking. Graph the function and interpret the domain, range, intercepts, symmetry, maximum, and minimum.

Lesson 2-4

Graph each absolute value equation.

27. $y = \left|x + \frac{1}{2}\right|$
28. $y = |x| + \frac{1}{4}$
29. $y = |4x - 3|$
30. $y = -|x + 4|$
31. $y = 2|x - 3|$
32. $y = |x + 4| - 2$

Lesson 2-4 *continued*

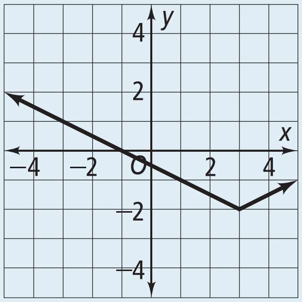
For Exercises 33–35, choose the graph that corresponds to each absolute value function.

33. $y = \frac{1}{4}|x|$

34. $y = |x + 3| + 2$

35. $y = \frac{1}{2}|x - 3| - 2$

A.



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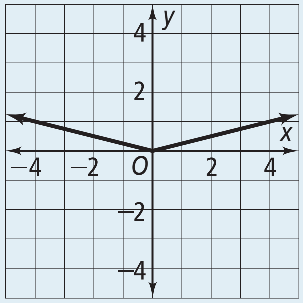

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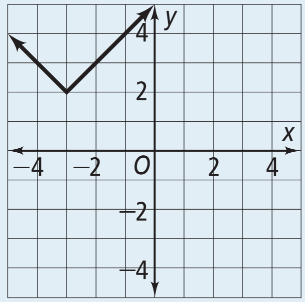
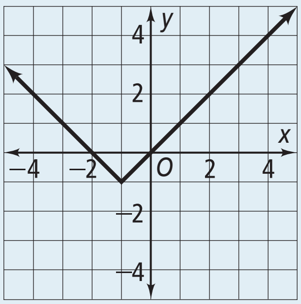
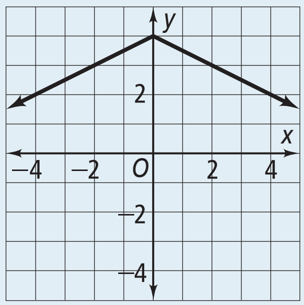


Write an absolute value function for each graph.

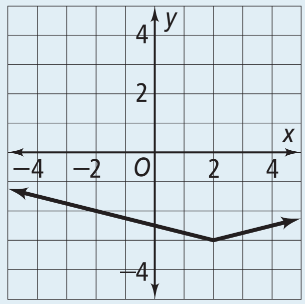
36.



37.



38.



Lesson 2-5

Graph each absolute value inequality.

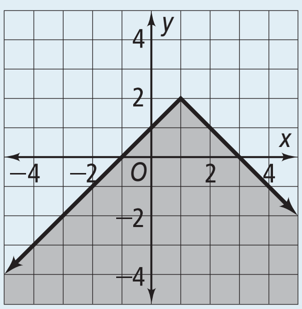
39. $y < |x - 1| - 2$

40. $y + 4 \geq |x + 1| + 5$

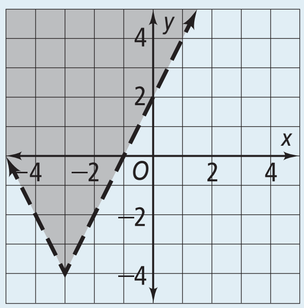
41. $0.5y + 1.5 \leq -0.25|x - 1|$

For Exercises 42–44, write the inequality that corresponds to each graph.

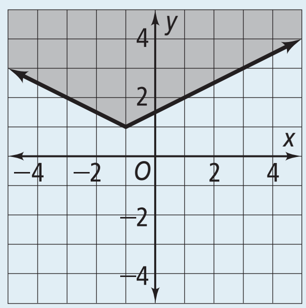
42.



43.



44.



Solve each absolute value inequality algebraically.

45. $y > \frac{1}{2}|x - 2| + 4$

46. $y \leq \left|\frac{x - 4}{3}\right| + 1$

47. $y \geq |3x - 3| + 1$