Study Guide Key

1) (a) $f(x) = (x-5)^2 - 4$

(b) D: $(-\infty, \infty)$ R: $[-4, \infty)$

(c) $(5, -4)$

(d) $x = h = 5$

(e) $(0, 21)$

(f) $(3,0)$, $(7,0)$

2) $f(x) = x^5 - 2x^4 + x^3 - 6x + 7 + \frac{-11}{x+2}$

3) $f(x) = x^4 - x^3 + x^2 - x + 1$

4) $f(x) = (x+1)(3x^3 + x^2 - 11x + 11) + 4$

5) $f(3-2i) = (3-2i)^2 - (3-2i) + 3 = 5 - 10i$

and

$$\begin{array}{r|rrr} 3-2i & 1 & -1 & 3 \\ & \downarrow & 3-2i & 2-10i \\ \hline & 1 & 2-2i & 5-10i \end{array}$$

6)

$$\begin{array}{r|rrrrr} \frac{1}{2} & 16 & 0 & 4 & 0 & -2 \\ & \downarrow & 8 & 4 & 4 & 2 \\ \hline & 16 & 8 & 8 & 4 & 0 \end{array}$$

$f(k) = f(\frac{1}{2}) = 0$, $k = \frac{1}{2}$ is a zero of $f(x)$.

7) $1\overline{)\begin{array}{ccccc} -3 & 1 & -5 & 2 & 4 \\ & -3 & -2 & -7 & -5 \\ \hline -3 & -2 & -7 & -5 & -1=r \end{array}}$

No, $x-1$ is not a factor of $f(x)$

8) $2x^2-x-6=f(x)$, so original function is $f(x)=(x-1)(2x+3)(x-2)$ where $k=1$

9) $x=\frac{1}{2}, -1\pm\frac{\sqrt{2}}{2}$

10) (a) $\pm1, \pm2, \pm4, \pm8$

(b) $-2, -1, 4$

(c) $f(x)=(x+2)(x+1)(x-4)$

11) $f(x)=-\frac{1}{2}x^3+2x^2+\frac{11}{2}x-15$

12) (a) 

(b) 

13) (a) $f(0) = 7 \rightarrow$ positive
and $f(1) = -1 \rightarrow$ negative

By I. M. V. T., there exists a real zero between 0 and 1

(b) $f(1) = -2 \rightarrow$ negative
and $f(2) = 6 \rightarrow$ positive

By I. M. V. T., there exists a real zero between 1 and 2

14) (a)
$$\begin{array}{r|ccccc} 2 & 1 & -1 & 3 & -8 & 8 \\ & \downarrow & 2 & 2 & 10 & 4 \\ \hline & 1 & 1 & 5 & 2 & 12 \end{array}$$
→ All positive and $K > 0$ so it's true

(b)
$$\begin{array}{r|ccccc} -2 & 1 & 1 & -1 & 0 & 3 \\ & \downarrow & -2 & 2 & -2 & 4 \\ \hline & 1 & -1 & 1 & -2 & 7 \end{array}$$
→ Alternating signs and $K < 0$, so it's true

15) (a) V.A.: $x = -2$

Oblique Asymptote: $y = x - 3$

(b) V.A.: $x = \pm 1$

H.A.: $y = 0$ or x-axis